LOCAL MODELLING OF THE FRACTAL DIMENSION ON DIGITAL ELEVATION MODELS

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Abstract
This study uses the box-counting method for local computation of the fractal dimension on a digital elevation model. The local method offers more detailed vertical ruggedness data than the global approach, providing deeper insight into surface structure. The work introduces an innovative local approach to determine the fractal dimension of surfaces and tests it on selected data samples. This work presents an effective geoscientific tool with potential cross-industry applications.

Keywords
Local fractal dimension, GIS, digital elevation model

1 INTRODUCTION

Fractal dimension (FD) serving as a quantitative indicator of similarity, density, complexity, or frequency is useful in the analysis of geomorphological phenomena, distribution patterns, and their comparisons [1], [2], [3]. The local approach to calculating the FD in the georelief can help detect various morphological features with higher precision providing a more detailed understanding of their complex structure [4]. This approach yields deeper insights into the shapes and patterns that constitute georelief and provides details about its structure that can be utilised in diverse industries (material structure analysis [5]), transportation (traffic network analysis [6], and optimising routes through areas with the least ruggedness), forest, agriculture [7], and others where knowledge of terrain surface flatness/ruggedness is important. This opens up new possibilities for surface analysis and interpretative procedures in geoscientific disciplines.

In recent years, the use of FD, and the broader concept of fractal geometry, has slowly gained recognition, especially in the study of objects and phenomena exhibiting fractal properties. Fractal geometry in this context deals with objects called fractals. A fractal is a scale-invariant object, meaning that fractal properties, such as FD, remain the same regardless of how large a part of the examined object is. Another property that follows from this is that fractals should be so-called self-similar objects, or they can also be self-affine, or statistically self-similar, as is the case with the Earth's surface [1], [8], [9], [10]. The Earth's surface possesses many properties, and from a geometric perspective, one of them is FD. There are several methods to determine FD [11], the box-counting method being among the most commonly used. The idea of the box-counting method is based on covering the fractal object with an n-dimensional grid of boxes, with their size determined for each iteration by the scaling factor r. The method determines the number of boxes N(r) needed to completely cover the object. FD, in this case more precisely referred to as box-counting dimension, is then obtained by analysing the relationship between the total number of boxes covering the object and the scaling factor (Equation 1).

$$D_B = \lim_{r \to \infty} \frac{\log(N(r))}{\log\left(\frac{1}{r}\right)} \tag{1}$$

Currently, FD is extensively applied in the field of geosciences to analyse spatial structures and phenomena to understand their geometric structure or behaviour [12], [13], [14], [15], [16], [17]. Its use provides valuable tools for a deeper understanding and description of geomorphological and geographical phenomena. This study focusses on describing the methodology, basic principles, procedures, and mathematical background of calculating the local fractal dimension (LFD) on a raster digital elevation model (DEM) using the box-counting method through an approach involving a moving computational window known as kernel.

2 METHODOLOGY

The box-counting dimension appears to be a very good and reliable indicator of the level of surface ruggedness of objects. In the case of 3D objects such as DEM, this indicator allows for a globally scaled assessment of its vertical ruggedness. However, a drawback of such a global approach lies in the generalisation of the ruggedness of individual surface parts, leading to a loss of information describing smaller surface fluctuations. The local approach to the FD calculation method enables the retrieval of this information and subsequently allows for a suitable interpretation of it in parts across the entire surface. The local FD calculation approach described in this work utilises the method of a moving computational window, referred to as kernel. The box-counting method is then applied within the kernel to obtain the local FD value in the central pixel of the kernel, progressively traversing the entire DEM raster. The procedure proposed in this work focusses on calculating LFD on a surface represented in the form of a raster. The development and testing of the local approach are carried out using the Python programming language.

Local approach to computing the fractal dimension of a raster surface

The determination of the FD using the box-counting method, as described above, is tied to the calculation limit where the size of the box approaches zero. To approximate this relationship for real-world structures and current computing capabilities, the calculation is performed through iterations up to a specific degree (based on the data density). Consequently, the resulting FD value is an estimate, determined as the slope of the regression line in a logarithmic (log-log) graph (a graph with logarithmic scale axes). The regression line approximates values from individual iterations, where log(r) values are plotted on the X-axis and log(N(r)) values on the Y-axis.

The local approach to computing the FD of a raster surface using the box-counting method has previously been applied (refer to [4]), with the calculation using the raster structure to estimate the dimensionality of the surface. This work focuses on the local approach to computing the FD of a surface, represented in raster form, through the method of a moving computational window. This approach treats the surface comprehensively as a 3D model of an irregular triangular network (TIN) generated by the Delaunay triangulation method [18]. Similarly, in the FD calculation, a Euclidean transformation of points is added to the kernel, enhancing the sensitivity to detect local surface fluctuations. The Euclidean transformation in the kernel employs translation and rotation of the system so that the XY plane aligns with the regression plane that best approximates the analysed points. The extent and dimensions of the kernel are defined by a square-shaped window containing the pixels of the analysed part of the surface raster. The size of the kernel can be adjusted to include even more distant pixels relative to the central pixel for which the fractal dimension is determined. It is important to note that enlarging the kernel window partially generalises the resulting FD by incorporating the ruggedness of the surrounding area into the calculation. The method of moving the computational window allows the kernel to gradually move across the entire raster model in orthogonal directions with a step size of 1 pixel. Finally, the central pixels of all kernels with calculated FD values are arranged into a separate raster along with their respective LFD values. The principle of the moving computational window is illustrated in Fig. 1.

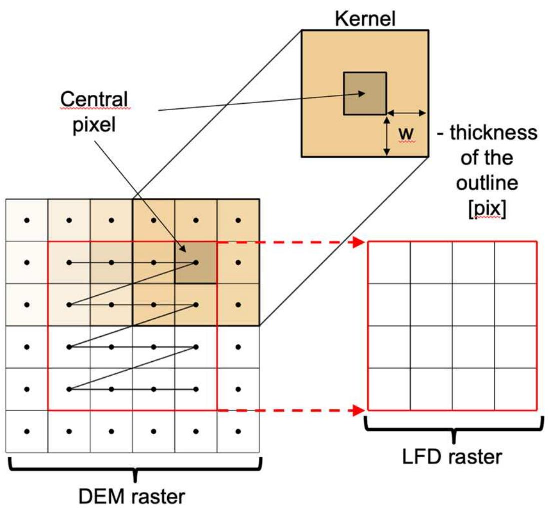

Fig. 1 The principle of the moving computational window.

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In the FD calculation, the main task for each iteration is to determine the exact number of boxes N(r) that geometrically overlap the TIN model. Geometric overlap of 2D/3D objects can be determined through an object collision detection algorithm, in this case, the analysed objects being a box and a triangle. The algorithm to detect collisions of objects is based on conditions defined by the Hyperplane separation theorem [19], [20], [21]. In the case of detecting the collision between a box and a triangle in 3D, testing is performed along a total of 13 potential separating axes, and the existence of at least one of them means that the objects do not overlap. This issue is explained in more detail in [22], [23]. Determining the values of N(r) for r, increasing geometrically (ensuring that the edge size of the box is halved with each iteration), and subsequently calculating the LFD using the kernel is described through the development diagram in Fig. 2.

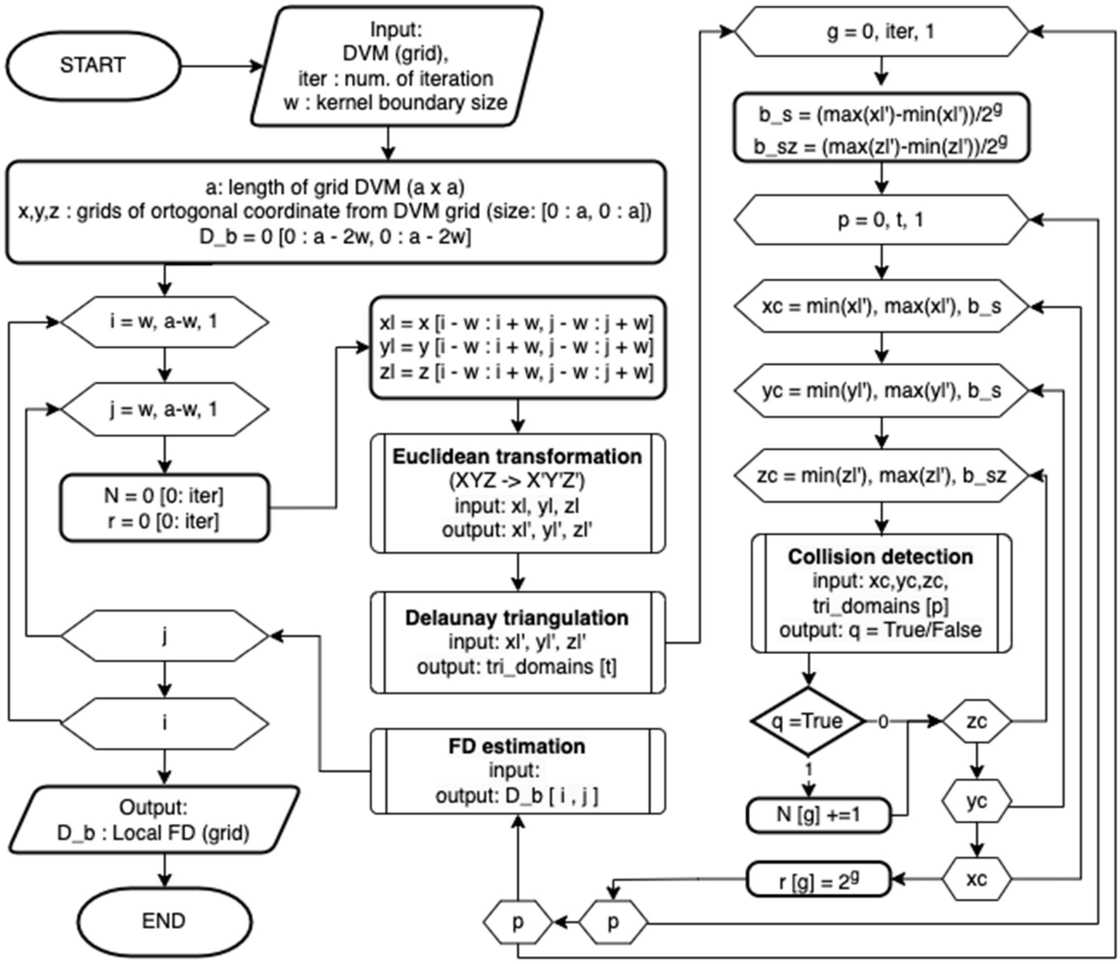

Fig. 2 Flowchart diagram for estimating local fractal dimension.

As depicted in Fig. 2, the input values for the algorithm include not only the digital vertical model (DVM) raster but also the parameters "iter" and "w". The "iter" parameter is chosen by the user as a whole number, defining the number of iterations performed when estimating the FD value. The "w" parameter, also a numerical value, defines the size of the kernel in the number of pixels in each direction surrounding the central pixel (thickness of the outline [pix]) of the kernel.

The steps outlined in the diagram were translated into an algorithm in Python language. Due to the high computational load on the processor, the algorithm was parallelised for computation on multiple logical cores of the processor. Parallelisation was implemented by dividing the computational domain into smaller segments that were then independently calculated in parallel. The results were subsequently merged back into the final output raster with the LFD values.

Input data

For the purpose of developing and subsequently testing the local approach to modelling the FD of surfaces, simple surfaces with different levels of ruggedness were created. These surfaces had a grid structure representing a raster. In total, 3 surfaces with an area of 50 × 50 m and a resolution of 1 m/pix were generated.

- The 1. surface is generated as an inclined plane defined by equation (2):

$$\frac{x}{2}+\frac{y}{2}-z=0 \tag{2}$$

- The 2. surface is generated by the function (3):

$$f(z)=20\sin\left(\frac{x}{3}\right)\cos\left(\frac{y}{6}\right)+20 \tag{3}$$

- The 3. surface is generated using the Brownian motion technique using Stein's method to simulate an artificial surface with a chosen Hurst parameter H = 0.9 in accordance [24].

All three simulation models of surfaces represent different types and degrees of height fluctuations, and thus also the ruggedness of the surfaces. Examples of these surfaces are shown in Fig. 3.

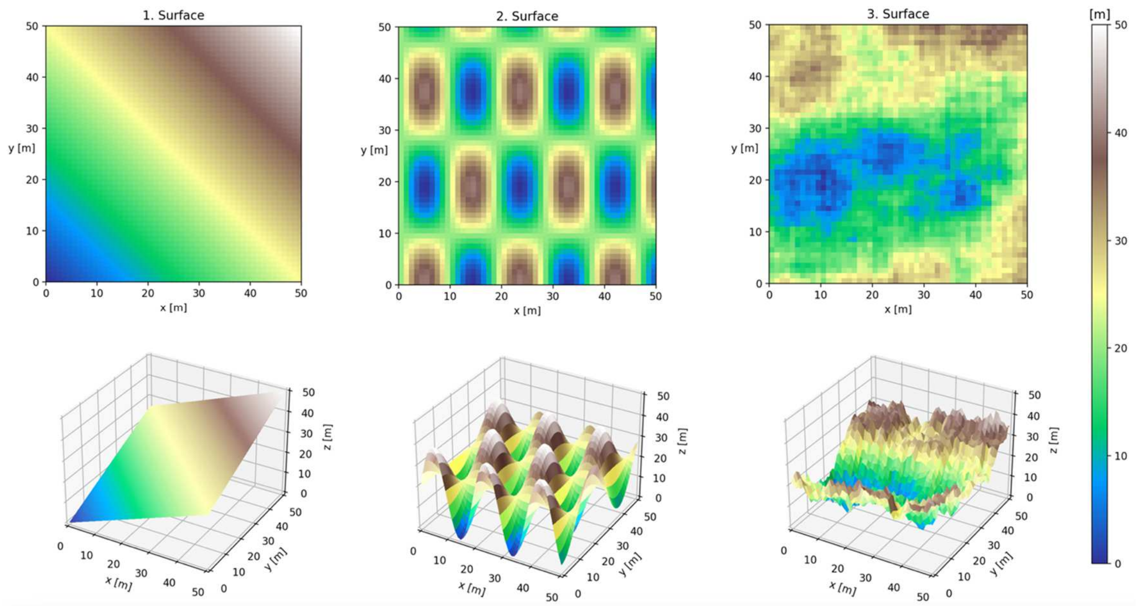

Fig. 3 Simulation surface models.

3 RESULTS

The local approach to calculating the FD on the DVM raster was tested on three simulation surfaces as described above. For these calculations, a kernel size of 5x5 pixels (w=2) was chosen. The total number of kernels for a 50 × 50 pixel area is 2116. To enhance calculation efficiency, parallelisation was proposed, in which the number of kernels was distributed to *n*-processes according to the number of CPU. The second optional parameter is the number of iterations to calculate the FD, which was set at 4 in the calculation. This number is optimal considering the density of points in the kernels and the need to demonstrate the functionality of the method. Under ideal conditions and with a sufficiently powerful processor, it is possible to increase the number of iterations to achieve a higher-quality result. The results of the calculated LFD values for all three surfaces, along with histograms expressing the distribution of the LFD values in the file, are visually represented for better interpretation in Fig. 4.

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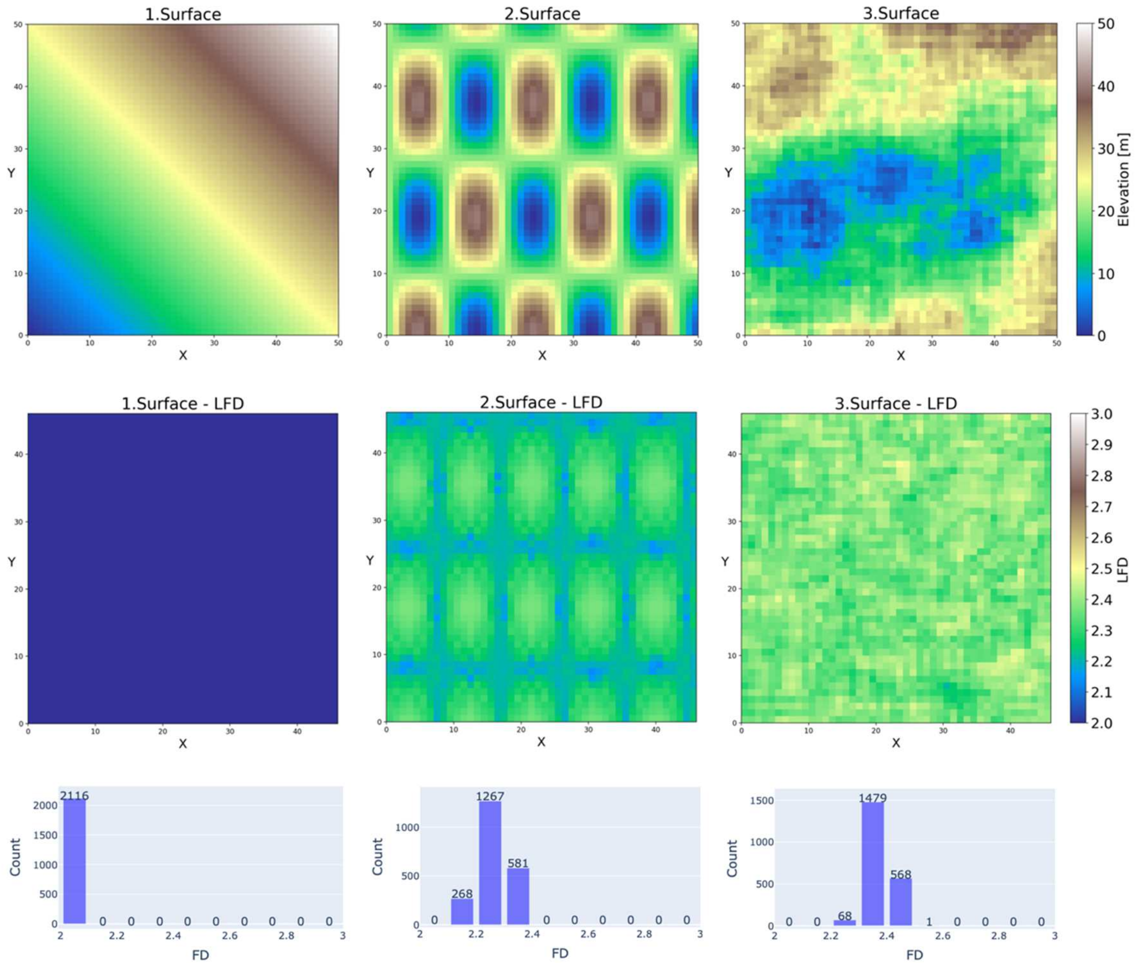

Fig. 4 Graphical interpretation of the resulting LFD on the simulation surface models.

The graphical representation of the LFD profiles of the surfaces, reveals how the value correlates with increasing/decreasing degrees of ruggedness in different parts of the surfaces. Further comparison, as stated in Tab. 1, numerically describes some statistical properties of the LFD on the simulation surfaces.

Tab. 1 Statistics of fractal properties of tested surfaces.

	1. Surface	**2. Surface**	**3. Surface**
global FD (4. iter)	*2.000*	*2.565*	*2.396*
min (LFD)	*2.000*	*2.134*	*2.212*
max (LFD)	*2.000*	*2.371*	*2.503*
mean (LFD)	*2.000*	*2.262*	*2.374*
med (LFD)	*2.000*	*2.258*	*2.375*

Tab. 1 displays the value of the global FD (4 calculation iterations) offering a better understanding of how the local approach differs from the result of the global FD. The comparison of the average LFD values of the surfaces shows that their ruggedness level is consistent with visual inspection. Despite the optimisation of the calculation, the computation time for LFD took 45–60 minutes using 12 logical cores of the computer's processor (Processor: 6-core Intel Core i7, RAM: 16GB, GPU: AMD Radeon Pro 5300M/Intel UHD Graphics 630).

4 DISCUSSION

FD is currently a significant quantitative indicator in the analysis of geomorphological phenomena and distribution patterns. This study focusses on the local approach to calculating the FD on DVM using the box-counting method through a moving computational window, referred to as a kernel. The results of this study indicate that the local approach to determining the fractal dimension can provide a more detailed view of the morphological features of surface relief with higher precision and detail. This approach has applications in various industries, including materials analysis [5], transportation [6], forestry, agriculture [7], and others where comprehensive understanding of the structure of the terrain surface is crucial or beneficial.

In experiments, this local approach was designed and tested on three different simulated surface models with varying levels of ruggedness. The obtained results suggest that the LFD values correlate with the increasing / decreasing degrees of surface ruggedness (Fig. 4). The values of the local approach to the calculation of FD were also compared with the values of the global approach, and it was observed that the local approach has the potential to interpret information about the spatial distribution of vertical irregularities of the surface in more detail compared to the global approach. This is due to the fact that unlike the global FD, which provides an overall view of vertical complexity, the local approach allows for obtaining more detailed and locally specific insights into variations in the terrain. One aspect of the LFD estimation algorithm that may be perceived negatively is its multilevel cycle of calculation (Fig. 2), which leads to increased computational time. The Python language, in which the tool was created and tested, is a high-level interpreted programming language with simple syntax, which is reflected in increased computational time. In this work, the Python language provided an optimal foundation for creating the tool due to its popularity among users. For future use, adapting the tool for individual purposes and needs, or rewriting it into another programming language with closer hardware interaction to accelerate computation, is a straightforward option. The approach described in this work expands current knowledge in the field of fractal surface analysis with new processing methods that offer more detailed results. These results have the potential for significant impact across various industries in the future.

5 CONCLUSION

This study introduced a methodology of local fractal dimension calculation on a DEM using the box-counting method and a moving computational window known as a kernel. This innovative approach allows for obtaining detailed information about the ruggedness of the surface, which has broad applications in geoscientific disciplines, as well as in other fields where knowledge of surface ruggedness intensity is required.

A dedicated tool for local FD calculation was developed, offering a means to determine the characteristics of the DEM in the analysis of surface ruggedness. It provides detailed information about the surface structure, which is important for various industries when using knowledge about the flatness and ruggedness of the terrain. This opens up new possibilities for a better understanding and description of geomorphological and geographical phenomena.

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