THROWSHED ANALYSIS BASED ON DIGITAL ELEVATION MODEL

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Abstract
Currently, computing impact areas that can be hit by any projectile is treated by using partial and generalised approaches. Therefore, the objective of this work is the development of automated procedures in the form of a freely available GIS tool programmed in Python language that uses the digital elevation model, physical parameters and settings to compute 'throwshed' - areas, which can be reached with any projectile shot from any given place. The archaeology of conflicts is considered an area of application, but the field of use may be larger. The paper deals with the very basics of a physical model for generating the results, which are also present in the work along with the discussion thereof. The presented results show many functionalities of the tool and offer a few use cases.

Keywords
Digital elevation model, geographic information system, throwshed, external ballistics

1 INTRODUCTION

The development of new and innovation of existing techniques for the collection of spatial data utilised in the creation of interpretations of the Earth's surface and terrain allows a continuous improvement of their resolution and precision. The digital terrain model (DTM) as one of the products of LiDAR (Light detection and ranging) or InSAR (Interferometric synthetic-aperture radar) data finds its applications in the delineation of geological structures in the field of geology, monitoring continental glaciers [1] and geological processes such as landslides [2] or deformative consequences of earthquakes [3] in the context of geodynamics. In the area of geographic information systems, the DTM is suitable for spatial analyses [4], [5]. One of the analyses is the visibility analysis, or viewshed, which defines areas visible from a given place and has applications in the field of urban planning [6] or archaeology [7].

Similar is the analysis under development that delineates areas that can be hit by a projectile shot from a specific place, an analysis also referred to as 'fireshed' [8] or 'throwshed'. The subject of this paper, in addition to the development of the analysis methodology, is also the design of a custom GIS programme in Python language that automates computation procedures. The archaeology of conflicts is considered to be the area of application, as the analysis could become a part of a broader KOCOA analysis that is used for battlefield reconstruction [9].

Current state of analysis and test data characteristics

Currently, archaeology of conflicts lacks an analysis that evaluates the vulnerability of the surrounding area to the impact of a projectile shot from a specified position as comprehensively as in the case of visibility analysis. Nowadays, reconstruction of historical battlefields, when defining the safety zone, that is, the region within the range of allied weaponry, typically treats the terrain as a simple flat plane with the generalisation of the entire analysis down to the definition of the maximum weapon range arc [10]. However, complex undulation of terrain and placing the firing location in one of the elevation extremes of the area significantly alter the range of the weapon, as well as the shape of the whole safety zone and the presence of terrain 'in defilade' (similarly to visibility analysis). Consequently, this can lead to an incorrectly interpreted affected area with further inappropriate implications. Several authors ([8], [11]) incorporated the terrain in the analysis in question; however, these attempts had many shortcomings or were very specific variations of the planned full-fledged throwshed analysis, which is yet to be developed to an extent similar to viewshed (in this context, a full-fledged refers to the analysis that assesses relevant curves that connect the position of the observer / shooter with each cell of the DTM raster).

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The test data refer to the product of the LiDAR mapping technique, specifically the DTM raster, with further information about the projectile and the firing location. This work features the product of the project of Airborne Laser Scanning (ALS) of the Slovak Republic, that took place in 2017 – 2023 [12], [13]. Specifically, this involves the fifth-generation DTM, which is emphasised due to its quality and accuracy. Due to its resolution of 1 × 1 m and its high vertical accuracy (in Bpv system), which does not exceed 10 cm in most collection lots, the DTM finds its applications in a wide range of areas [12]. With a high average of the last reflection point density over 15 lrp/m^2 and in some lots even exceeding 20 lrp/m^2, it is possible to generate a DTM with 0.5 × 0.5 m or 0.25 × 0.25 m resolution from the original point clouds. Such a high-resolution DTM is used within the field of cultural heritage preservation, where the advantage of detail and product quality outweighs the issue of quadratically increasing data size, which can be averted anyway by appropriate compression of files containing data. Another advantage is that the point clouds as well as the DTM 5.0 are provided free of charge by the map client of ZBGIS [14], although they are subject to copyright, which has to be respected when using and presenting the data. It is also important to note the second cycle of Airborne Laser Scanning in Slovakia, which, starting in 2022, will update ALS products (creating DTM 6.0) [15].

2 METHODOLOGY

The tool created for the throwshed analysis consists of several algorithms, some of which are described and divided into the following subsections. Algorithms are part of the programme, that was developed in the Python 3 programming language. The programme is publicly accessible at [16].

Physical model of generated trajectory

To make the throwshed analysis comparable to visibility analysis, the curves connecting the starting and target cells of the DTM raster must be compared with terrain profiles between the cells. In the context of throwshed analysis, the curves are ballistic trajectories for which there are several ways of calculation. In analytical methods, one can encounter a parabolic definition of trajectory respected by a mass point in a vacuum, thus without any influence of forces acting on the body due to aerodynamic drag [17]. Equation (1) describes the mathematical formula of the parabola:

$$Y = X * tan\Phi_0 - \frac{gX^2}{2V_0^2 cos^2\Phi_0} \tag{1}$$

where X, Y are the coordinates in the vertical plane of the trajectory in [m], Φ_0 is the shooting angle in °, g is the magnitude of gravitational acceleration in m/s^2 and V_0 is the initial (muzzle) velocity of the projectile in m/s. Limiting the model only to gravitational force can be useful for simple tasks at short ranges; however, in the context of this analysis, it would be a solution with a significant degree of approximation.

The work [17] also describes analytical methods for calculating flat-fire trajectories, in which some approximations are introduced, namely the dominant effect of the velocity vector in the direction of the X-axis in the vertical plane of the trajectory with the origin at the point where the projectile leaves the weapon (transition from internal to external ballistics) as it passes the local horizon. Typical is the Siacci method [18], still used in sports today. Equation (2) for calculating the approximate trajectory incorporates the ballistic coefficient C in kg/m^2 and the initial height Y_0 in m, along with tabulated, dimensionless values of spatial $S(V)$, altitude $A(V)$ and inclination $I(V)$ functions computed for specific types of main projectiles G1–G8 at specific projectile velocities.

$$Y = Y_0 + X * tan\Phi_0 - \frac{1}{2}CX\left[\frac{A(V) - A(V_0)}{S(V) - S(V_0)} - I(V_0)\right] \tag{2}$$

Despite the very accurate representation of the real ballistic curve, using this method would be highly limiting due to the maximum slope constraint upon impact of 15°. The analysis requires a solution for trajectories that far exceed this limitation, ideally with a full range of <-90°, 90°>.

The algorithm for calculating trajectories is currently based on the Euler numerical method, also known as the 'first-order' integration scheme. In this method, the whole trajectory is divided into numerous small elements that are sequentially computed from the shooting point. The calculation involves parameters, such as the shooting angle, air density, magnitude of gravitational acceleration, initial velocity, mass, the cross-sectional area of the projectile and the aerodynamic drag coefficient. Additionally, gravitational acceleration acts in the vertical direction along the Y-axis. The calculation also considers the deceleration due to the drag force, which acts in the direction opposite to the projectile velocity vector. This deceleration needs to be divided into X and Y components.

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The result in each integration element is a coordinate difference between the left and right points of the element, which is used to compute the new trajectory point, where time *t* serves as an independent variable.

Model for generating impact areas

In visibility analysis, curves and lines of sight connecting the viewpoint and the target cell are generated based on relative elevation and horizontal distance, from which the mutual slope is computed. The slope is then compared with the slope of the line of sight to each intermediate cell in the terrain profile, and if the slope to the target cell is not exceeded, the target cell is considered visible [19]. In throwshed analysis, the slope cannot be used, so it is necessary to compare the heights of intermediate cells with the corresponding heights of the trajectory leading to the target cell.

Currently, the programme seeks a trajectory intersecting the target cell iteratively by gradual approaching. By changing the values of the input parameters, many combinations can be found, where it is possible to intersect the cell. However, from a practical standpoint, it is most convenient to only modify the values of the shooting angle while keeping all other parameters fixed. This approach allows the cell, if it is within range, to be impacted twice [20], once by a trajectory with a higher shooting angle (utilised in artilleries) and once by a trajectory with a lower shooting angle (directly). With different settings for the shooting angle range, it is then possible to hit the cell within the range 0-2 times.

If the trajectory network is generated over the full range of the shooting angle, its envelope rotated around the shooting point defines a safety zone [21], represented by a paraboloid in the case of parabolic trajectories. However, with complex ballistic curves, more intricate shapes are formed. An example of the envelope is shown in Fig. 1. Due to the definition of such a shape, the entire area in question is divided into potentially reachable and never reachable regions, which accelerates the evaluation process. Within the former region, the trajectories intersecting the target cells are evaluated, to decide whether any of the intermediate cells from the terrain profile exceed the trajectory, which determines the accessibility of the cell. None of the DTM cells remain overlooked and it is possible to even delineate areas covered by obstructions, which means the cells within the safety zone that cannot be hit.

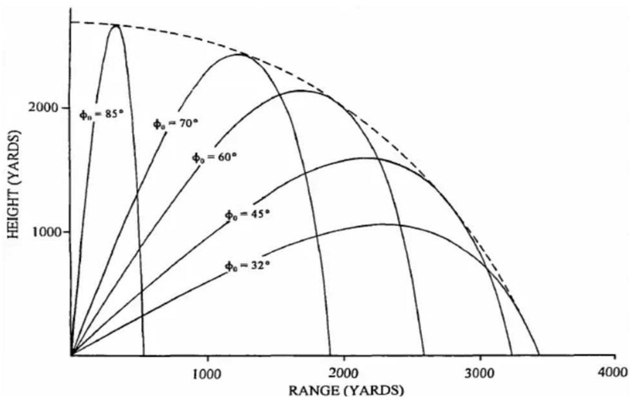

Fig. 1 Envelope of the ballistic trajectory network (dashed line) [1].

The result of the analysis is a georeferenced raster with cell values from 1 to *n*, where *n* represents the number of impacts from multiple shooting points. The raster consists of two bands, which corresponds to the evaluation outcome of both types of intersecting trajectories. Band 1 provides information about impacts from trajectories with a lower shooting angle, while band 2 corresponds to the higher one. The programme offers several calculation modes that can be combined. Practical examples of the results are presented in Chapter 4.

3 RESULTS

At present, the programme producing the throwshed offers few functions. For all cases, a location within Slovakia was selected, and the DTM with a resolution of 1 × 1 m was obtained from the website of ÚGKK SR [14]. It is a simple terrain with an elevated area in the middle, that will serve as a shooting ground for 1 and 5 shooters. The DTM is shown in Fig. 2. The projectile used is a simple spherical stone for a slingshot with a 5 cm diameter. Tab. 1 contains the input parameters.

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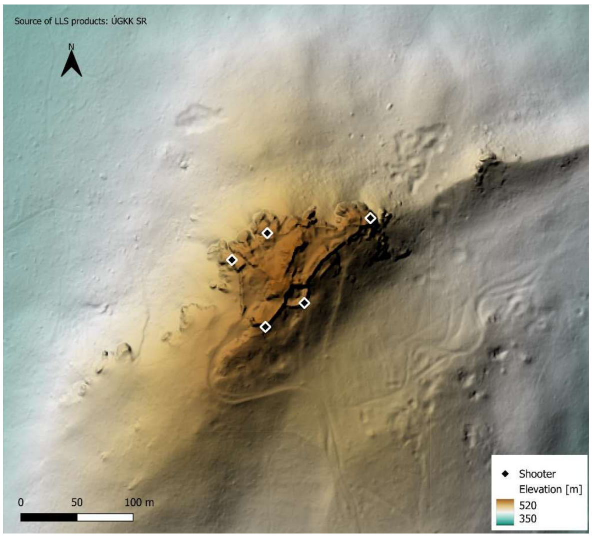

Fig. 2 DTM of selected test location.

Tab. 1 Input parameters.

Initial height above ground [m]	1.700
Shooting angle range [°]	<-90; 15>
Gravitational acceleration [m/s^2]	9.810
Initial velocity [m/s]	50.000
Air density [kg/m^3]	1.225
Mass [kg]	0.100
Cross-sectional area [m^2]	0.001963
Drag coefficient	0.47

The first setting involves a simple assessment of the presence of DTM cells within the safety zone (labelled 'simple throwshed'). Fig. 3 on the left shows the terrain coverage by areas that can be hit from one shooting location (1 shooter) without considering the inaccessible regions. Band 1 is presented. With the range of the shooting angle up to 15°, there are no second intersecting trajectories with a higher shooting angle, which is why there is no band 2 shown for this and following cases (except for the last one). In the following setting, the number of shooters was increased to 5. Fig. 3 on the right contains the result of the analysis, which shows a larger coverage of the area.

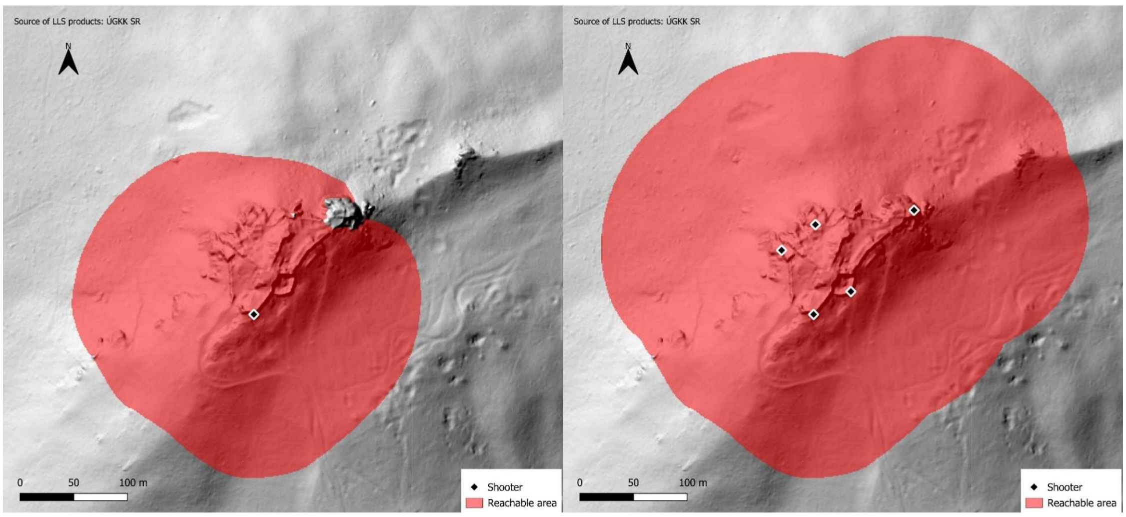

Fig. 3 Simple throwshed: safety zone, on the left: 1 shooter case, on the right: case of 5 shooters.

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On the left of Fig. 4, a similar case for five shooters is displayed, although in cumulative mode. The raster result presents the number of overlaps of all simple throwsheds generated for each shooter. On the right of Fig. 4, the analysis result for 1 shooter clipped by the visibility analysis raster can be seen.

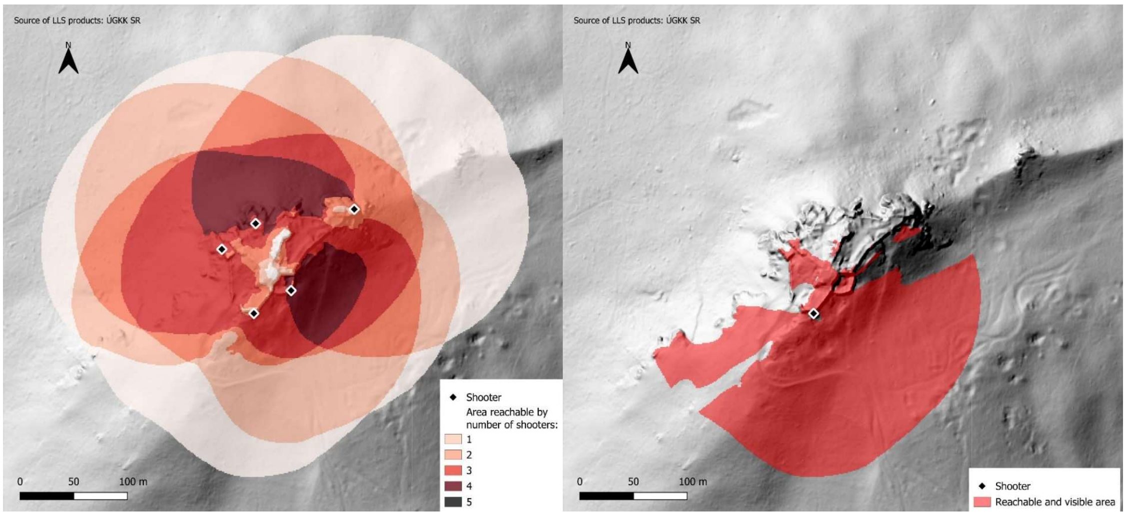

Fig. 4 Simple throwshed: safety zone, on the left: cumulative mode with 5 shooters, on the right: throwshed for 1 shooter clipped by viewshed.

The next raster shown on the left of Fig. 5 is the result of the assessment of the trajectories that intersect target cells within the safety zone compared to the terrain profiles, thus areas 'in defilade' are respected (labelled 'classic throwshed'). On the right of the same figure, a similar result for five shooters in cumulative mode can be seen.

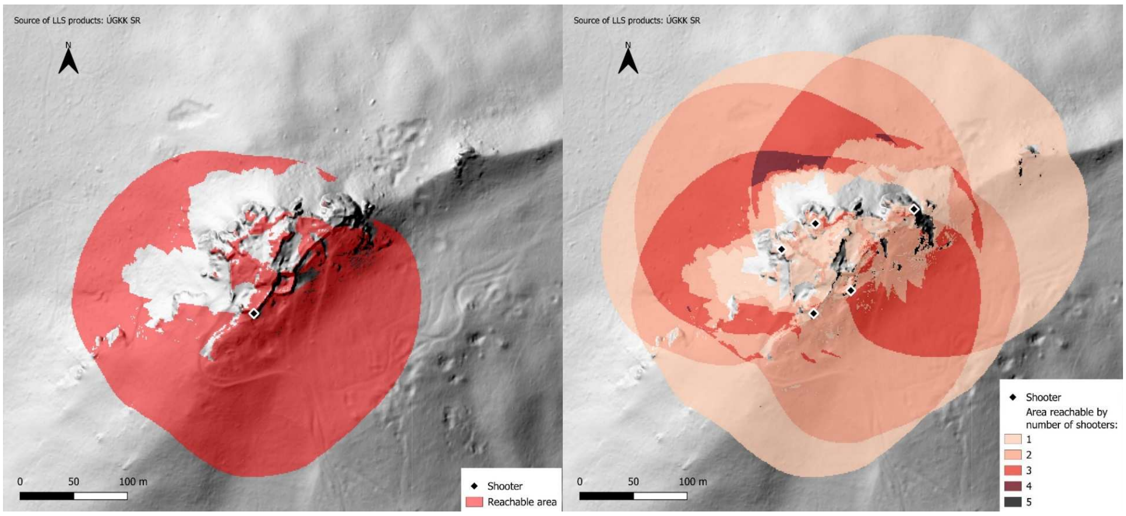

Fig. 5 Classic throwshed: on the left: 1 shooter, on the right: 5 shooters in cumulative mode.

The tool allows for the use of a line vector layer, which serves as an obstacle (a wall). The result incorporating a 4-metre-high obstacle is shown on the left of Fig. 6. On the right of the figure, the result of the classic throwshed is shown, where the shooting angle range was set to the maximum, 90°. Therefore, both bands are displayed because trajectories with higher shooting angles were evaluated as well. A larger part of the accessible area can be reached by both types of trajectories; the smaller one can be hit by high shooting angle trajectories only.

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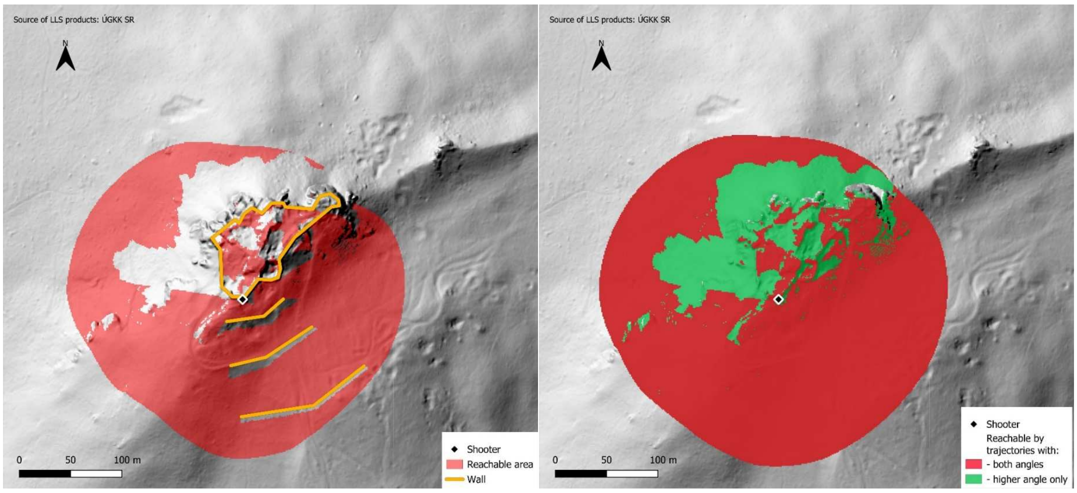

Fig. 6 Classic throwshed: on the left: utilisation of lines as obstacles, on the right: both bands.

4 DISCUSSION

Diverse results from the previous chapter can be associated with a number of use cases. The first two examples, where simple coverage of the safety zone was determined, were the least time-consuming. Comparing the cumulative mode cases with five shooters from simple and classic throwshed shows that it is impossible to hit some areas in close vicinity of the shooting positions, even though they are located within the safety zone, and the density of overlaps was reduced. In the case of using visibility analysis, the results can be interpreted as ‚I shoot at what I see‘. Comparing this result with the classic throwshed case for 1 shooter proves that areas invisible to a shooter may still be reached by projectiles fired by the same shooter. In the case of line utilisation as obstacles, the difference is clear, and this feature could be useful for tasks where the height and position of non-existent (long gone or planned) fortification is known. The last case serves as evidence that areas inaccessible by low-shooting angle trajectories can be reached by projectiles shot at a higher angle.

5 CONCLUSION

Although throwshed analysis is still in its early stages of development, the provided examples demonstrate that the tool under development offers several useful functionalities. The potential of application is primarily seen in the field of archaeology of conflicts for the reconstruction of battlefields, though the scope of use could be much broader. The results provided some use cases. From a result visualisation perspective, it remains a question of how two bands in a cumulative setting could be displayed, making the number of cell impacts clear for both cases of intersecting trajectories (the example featured only one shooter which simplified the interpretation).

There are plans to add functions that incorporate the accuracy of the result, consider the wobble motion of an arrow released from a bow, as well as changing the independent variable from time t to the range X in the numerical method. It will also be necessary to test the suitability of other numerical methods, namely Heun or Runge-Kutta in few orders [22], the implementation of the physical model for projectiles with velocities above the speed of sound, and the optimisation of the programme code, for example, by using parallel computation.

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