SLAB-COLUMN CONNECTION OF CROSS-LAMINATED TIMBER FOR PUNCHING: COMPARISON OF CURRENT SIMPLIFIED METHODS

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Abstract
This publication compares simplified analytical methods for calculating punching shear resistance in the slab-column connection of cross-laminated timber (CLT). The work compares simplified models according to Mestek and Muster. The reference experiment was taken from Mestek's dissertation thesis. By comparing these analytical models, it was demonstrated that none of the analytical models could accurately predict the failure of the specimen due to rolling shear, and lead to uneconomical design.

Keywords
Cross-Laminated timber, point support, concentrated loading, rolling shear

1 INTRODUCTION

In recent years, the development of structures made from cross-laminated timber (CLT) has brought about innovative solutions for ceiling structures in this type of construction, particularly point-supported slabs. However, such solutions have introduced previously unexplored methods of loading of individual lamellas in the slab-column connection. Since 2010, several publications have addressed various aspects of this issue, such as simplified analytical models [1], [2], reinforcement of the slab-column connection using screws [1], [3], [4], reinforcement of the CLT through the thickening of the plate cross-section above the support [5], and even projections of potential future solutions for ceiling structures using butt-glued connections of CLT panels [6]. In the future, further experiments in this field are planned under the research program at Fast+Epp in collaboration with the University of Northern British Columbia, where they are looking to provide inputs for point supported CLT [7].

One of the significant implementations using the mentioned structural system is the Brock Commons Tallwood House, a student residence at the University of British Columbia in Canada, built in 2016. This project involved experiments at a real scale to verify the design of cross-laminated timber.

In such a construction solution, shear resistance dominates over flexural resistance, deflection, and vibration of the ceiling structure, as seen in line-supported slabs. According to Muster [2], failure in rolling shear is neither ductile nor brittle. If a lamella is locally subjected to rolling shear, nonlinear deformations before failure allow the redistribution of stresses to lamellas that were not previously loaded. The strength of CLT in shear is determined by the rolling shear strength in the lamella perpendicular to the direction of loading [7]. The mode of failure is shown on Fig. 1.

However, existing simplified analytical models have inconsistencies, which this article addresses through a direct comparison. This comparison exclusively focuses on shear resistance while neglecting flexural or perpendicular-to-grain compression resistance.

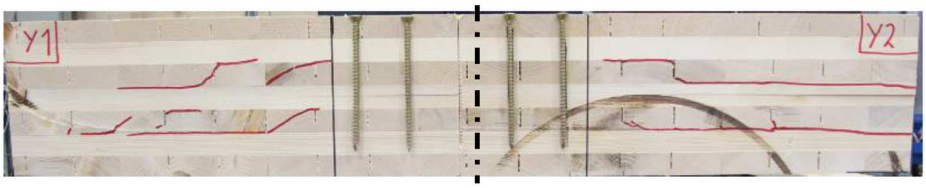

Fig. 1 Rolling shear failure in punching experiment [1].

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2 METHODOLOGY

Analysis of punching resistance in the slab-column connection of locally supported slabs bearing in two directions

The analysis of shear stress can be divided into three fundamental issues:

- Distribution of shear forces into the support.
- Calculation of shear stress.
- Calculation of punching resistance.

How individual analytical models approach these issues is described in the following chapters.

The first simplified method for determining the punching resistance was introduced by Mestek [1] in his dissertation thesis in 2011. It is based on the Shear Analogy Method, which is outlined in Appendix D of DIN 1052 standard.

Another examined model is the analytical model proposed by Muster [2], who also presented it in his dissertation thesis in 2020.

Both analytical models are based on the assumption that the distribution of load from the edge of the support occurs at an angle of 35°. Additionally, the critical shear path is located at the intersection of the centreline of the CLT panel and the intersection of this diagonal, as shown in Fig. 2 and Fig. 3.

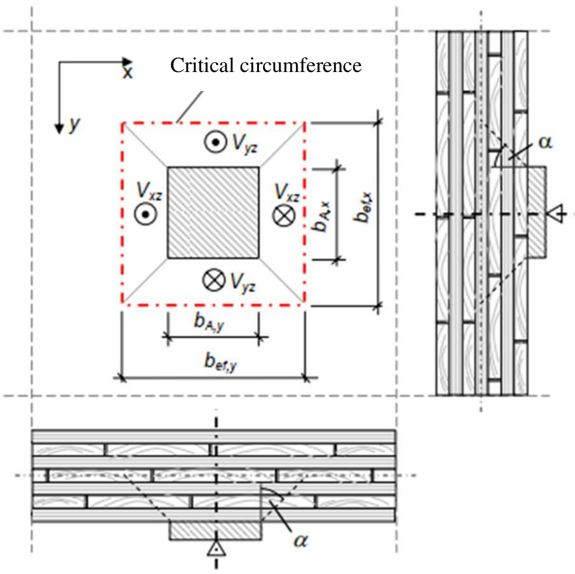

Fig. 2 Critical circumference [1].

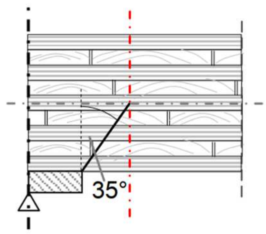

Fig. 3 Angle of load distribution [1].

Distribution of shear force into the support

In his doctoral thesis, Mestek relies on the Shear Analogy Method, based on which he created a rod model. The rod model consists of a grid of rods in two directions, *X* and *Y*, with a spacing of 100 mm at two levels, *A* and *B*. The rod stiffness is calculated based on Appendix D.3 of DIN 1052 standard, considering material properties for C24 timber. The model is loaded at its centre with a force *F*, and the distribution of forces on each face of the support is subsequently determined from the sum of shear forces, $V_{iz,A}$ and $V_{iz,B}$. Based on the results of this parametric study, regression curves are evaluated, describing the distribution in the main load direction according to Equations (1), (2), (3).

For the ratio of the lamella thickness in the *x*-direction (d_x) to the lamella thickness in the y-direction (d_y) $d_x / d_y = 1.0$

$$V_{XZ} \approx 0.33 \times n^{-0,1}.F \tag{1}$$

for $dx / dy = 1.5$

$$V_{XZ} \approx 0.39 \times n^{-0,1}.F \tag{2}$$

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for $dx / dy = 2.0$

$$V_{XZ} \approx 0.42 \times n^{-0.1} \times F \tag{3}$$

where V_{xz} is a shear force in kN, n is number of layers of CLT and F is total force reaction in column in kN. According to Mestek, the flow of shear forces is predominantly dependent on the ratio of lamella thickness in the x-direction to the y-direction, as well as the number of layers in the panel.

The study is conducted on samples with the following limiting factors:

- Total sample thickness (d): $0.10 < d < 0.22$ meters.
- Ratio of external dimensions of the sample (l / b): $1 < l / b < 3$.
- Number of layers (n): $5 < n < 11$.

These constraints define the range of values within which the study is conducted, ensuring that the analysis covers a variety of sample configurations and lamella thickness ratios.

Muster, in determining the forces on the face of the support, combines the strip method by Hillerborg and the beam network theory by Homberg. The analytical model expresses the coefficients β_x (4) and β_y (5), which define the proportion of the shear force above in the support in the x and y directions of the local coordinate system.

$$\beta_X = \frac{1}{2} \times \frac{l_y}{l_x} \times \left(\frac{1}{2} + \frac{EI_x}{EI_x + EI_y}\right) \tag{4}$$

$$\beta_y = 1 - \beta_X \tag{5}$$

where β_x is the shear force factor for direction x, β_y is the shear force factor for direction x, l_y and l_x is the span in x and y directions in m, EI_x and EI_y is the bending stiffness in direction x and y GPa×m^4.

The shear force in the main load direction is calculated according to Equation (6). V_{xz} represents the sum of shear forces on both faces of the support in the x-direction of the local coordinate system of the panel.

$$V_{XZ} = \beta_X \times F \tag{6}$$

where V_{xz} is the shear force in direction x in kN, β_x is shear force factor for direction x and F is total force reaction in the column in kN. For the central column of the column network with a regular grid having l_x along l_y, the shear force in the main load direction on one of the faces of the support is equal to half of V_{xz}. The procedure for calculating the shear forces above the other columns is described in detail in [2].

In Muster's model, it can be observed that the flow of shear forces depends on the flexural stiffness and spans in the two main directions. This calculation method is applicable under the assumption of a uniform column grid, where the ratio of spans cannot exceed 1.6, and the ratio of stiffness should fall within the range of 0.2 to 5.1.

Calculation of shear stress

Mestek states that in the case of cross-laminated timber without bonding of the shorter edges of the lamellae, the shear stress in rolling shear depends only on the shear force in plane B according to the shear analogy method. In Appendix D.3 of DIN 1052, Equation (7) provides the shear stress in rolling shear in the main load direction. However, this equation offers precise solutions only for 3- and 5-layer CLT panels. According to Equation (7), $v_{b,xz}$ is the shear force per meter of length, which is divided by the effective height a, where the effective height is the distance between the centres of the outer lamellae (valid only for 3- and 5-layer panels) effective in shear. Mestek rewrote this equation in the form of Equation (8), where the shear force V_{iz} is transferred through the effective width $b_{i,eff}$, the sum of which is the critical circumference. The effective section height is calculated based on the sum of the lamella thickness in the x and y directions, multiplied by the factor $k_{R,i}$, where $\tau R_{,xz\_D3}$ is the shear stress in MPa, $vB_{,xz}$ is shear force in kN/m, and a is the effective height of the cross section in m. $\tau R_{,iz}$ is the shear stress in *i-direction* in MPa, *Viz* is shear force in *i-direction* in kN, $kA_{,i}$ is a factor for edge columns, $kA_{,i}$ factor for effective height, $bi_{,eff}$ is the effective width of the support face in i-direction in m, dx and dy *is* the thickness of a single layer in x and y direction in m.

Tab. 1. The factor $k_{A,i}$ applies only to columns located on the edge of the slab, where this factor adjusts the average stress value on the effective width $b_{i,eff}$ to the maximum shear stress value due to the non-uniform distribution of shear stress over these columns. The values of the factor $k_{A,i}$ are provided in Tab. 2.

$$\tau_{R,xz\_D3} = \frac{v_{B,xz}}{a} \tag{7}$$

$$\tau_{R,iz} = \frac{V_{iz} \times k_{A,i}}{k_{R,i} \times b_{i.eff} \times (d_x + d_y)}, \tag{8}$$

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where τ_{R,xz_D3} is the shear stress in MPa, $v_{B,xz}$ is shear force in kN/m, and a is the effective height of the cross section in m. $\tau_{R,iz}$ is the shear stress in i-direction in MPa, V_{iz} is shear force in i-direction in kN, $k_{A,i}$ is a factor for edge columns, $k_{A,i}$ factor for effective height, $b_{i,eff}$ is the effective width of the support face in i-direction in m, d_x and d_y is the thickness of a single layer in x and y direction in m.

Tab. 1 Factor $k_{R,i}$.

Factor	**Number of lamellas**			
$\mathbf{k_{R,i}}$	**5**	**7**	**9**	**11**
$\mathbf{k_{R,x}}$	2.00	2.50	3.33	3.89
$\mathbf{k_{R,y}}$	1.00	2.00	2.50	3.33

Tab. 2 Factor $k_{A,i}$.

Factor $\mathbf{k_{A,i}}$	**The ratio of the width of the support in the direction i to the thickness of the panel** $\mathbf{b_{A,i} / d}$		
	1.0	1.5	2.0
$\mathbf{k_{A,i}}$	1.35	1.5	1.65

where i are local coordinates of CLT slab x and y

Muster's model involves calculating shear stress based on a parabolic stress distribution in a square homogeneous section, as described by Equation (9). Here, F_d is the support force, and t_{CLT} is the total thickness of the CLT panel. Equation (9) is applicable only when the ratio of spans and the ratio of flexural stiffness is close to 1. In other cases, the equation needs to be supplemented with the coefficients β_x and β_y.

For shear stress on any face of the support, Equation (10) is valid. V_{iz} is the force on the face of the support according to Muster's model for shear force distribution. The equation is supplemented by the factor k_{edge}, which considers the presence of an opening in the CLT panel directly above the column and is described in more detail in [2]

$$\tau_{R,xz} = \frac{1.5 \times F}{2.\left(b_{x,eff} + b_{y,eff}\right) t_{CLT}} \tag{9}$$

$$\tau_{R,iz} = \frac{1.5 \times V_{iz} \times k_{A,i} \times k_{edge}}{b_{i.eff} \times t_{CLT}} \tag{10}$$

where $\tau_{R,iz}$ is the shear stress in i-direction in MPa, F is total force reaction in the column in kN. $b_{i,eff}$ is the effective width of the support face in i-direction in m, V_{iz} is shear force in i-direction in kN and t_{CLT} is the thickness of CLT slab in m.

Calculation of punching resistance

According to Mestek's model, the 5th percentile of shear strength of timber in rolling shear can be multiplied by the factor $k_{r,90}$, which is used in Equations (11) and (12). This factor takes into account the increase in shear strength due to perpendicular-to-grain pressure stress $\sigma_{c,90}$

$$\tau_{R,xz,Rd} = \frac{k_{mod} \times k_{r,90} \times f_{R,k}}{\gamma_M} \tag{11}$$

$$k_{r,90} = min\begin{Bmatrix} 1 + 0.35 \times \sigma_{c,90} \\ 1.20 \end{Bmatrix} \tag{12}$$

where $\tau_{R,xz,Rd}$ is the rolling shear strength in MPa, $k_{r,90}$ is the strengthening factor for rolling shear strength, $f_{R,k}$ is the characteristic value of the rolling shear strength in MPa, k_{mod} is modification factor, γ_M is the partial factor, $\sigma_{c,90}$ is the stress perpendicular to grain in MPa.

According to Muster's model, the 5th percentile of shear strength of timber in rolling shear can be multiplied by the factor $k_{r,pu}$ = 1.60 (13) if the cross-laminated timber is subjected to concentrated loading, as is the case with point-supported slabs. However, for the case of an edge column, $k_{r,pu}$ should be limited to 1.30

$$\tau_{R,xz,Rd} = \frac{k_{mod} \times k_{r,pu} \times f_{R,k}}{\gamma_M} \tag{13}$$

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where $\tau_{R,xz,Rd}$ is the rolling shear strength in MPa, $k_{r,pu}$ is the strengthening factor for rolling shear strength, $f_{R,k}$ is the characteristic value of the rolling shear strength in MPa, k_{mod} is the modification factor, γ_M is the partial factor.

Comparison of analytical models

For the purpose of comparison between simplified methods, a calculation was performed that compares the resistance of CLT panels in punching when reaching the failure point according to the experiments in Mestek's doctoral thesis [1], which serves as the reference value for the comparison, as shown in **Chyba! Nenalezen zdroj odkazů.** and **Chyba! Nenalezen zdroj odkazů.**.

The first and second methods, M1 and M2, have been discussed in previous chapters. The calculation is complemented by an additional computational procedure, M3, which respects Muster's force distribution but calculates the shear stress on the effective section based on Grasho''s formula (14). The final method, M4, is a combination of methods M1 and M2, where the force distribution is taken from Muster's model M2, and the calculation of shear stress and resistance is based on model M1.

$$\tau_{iz} = \frac{V_{iz} \times S_{iy}}{I_{iy.} \times b_{(z)}} \tag{14}$$

where τ_{iz} is the shear stress in kPa, V_{iz} is the shear force in kN, S_{iy} is the static moment in m^3, I_{iy} is the moment of intertia in m^4, $b_{(z)}$ is the effective width in m.

Four analytical methods will be compared

- M1 – Analytical model according to Mestek.
- M2 – Analytical model according to Muster.
- M3 – Analytical model according to Muster with shear stress according to Grashof.
- M4 – Analytical model according to Muster with shear stress according to Mestek.

The comparison is conducted on a 5-layer CLT panel sample with lamella thickness of 27 mm and a total thickness of 189 mm. The average rolling shear strength of the timber in the panel is $f_{Rv,mean}$ = 0.95 MPa. The panel has overall dimensions of 1.46 m by 1.1 m (x-direction by y-direction), and the sample is supported around its perimeter and compressed in the centre with a base plate of 300 mm by 300 mm. Three samples were tested in total, with an average failure value of F_{mean} = 381.12 kN.

The samples are reinforced against perpendicular-to-grain pressure to prevent failure by this means.

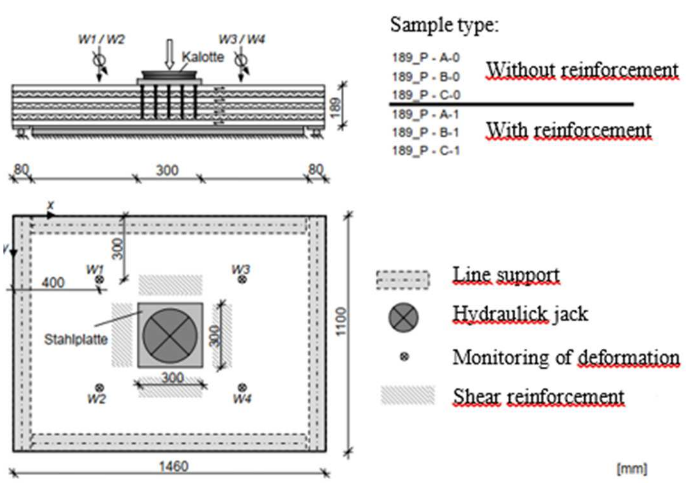

Fig. 4 Setup of experiment [1].

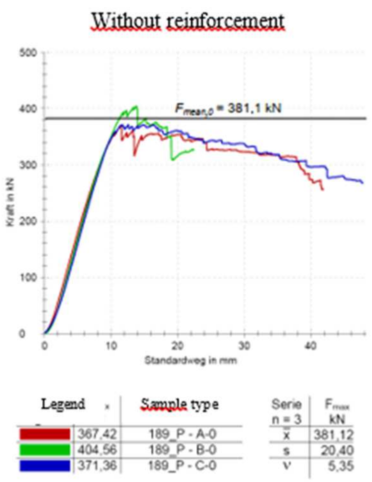

Fig. 5 Loading diagram [1].

Additional FEM analysis was performed to validate the distribution of shear forces to supports. The model was created in SOFiSTiK. The orthogonal anisotropy was considered with layered material option in software. The model was loaded with Fmean = 381.12 kN, and a linear analysis was performed. Sums of support reactions were observed, see Fig. 6.

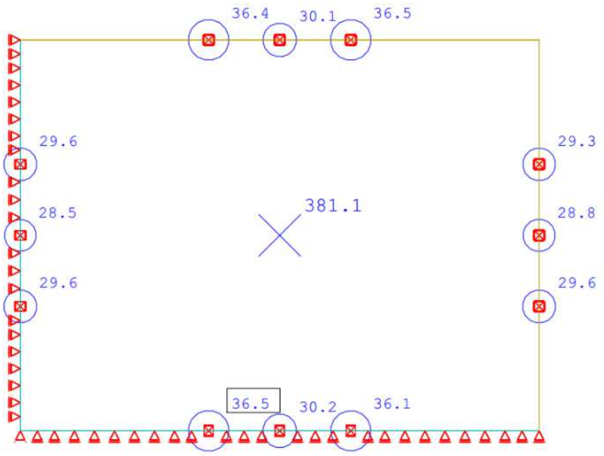

Fig. 6 Simple FEM model to validate distribution of shear force.

3 RESULTS

Tab. 3 summarizes the results of the different methods, where V_{Riz} is the shear resistance in the corresponding direction (the sum of resistances of opposite faces of the support), V_{iz} is the shear force in the respective directions when loaded with a force of F_{mean}. Shear stresses τ_{iz} are the stresses in rolling shear induced by the force on the corresponding face of the support at the critical layer of the panel. Fig. 7 shows the shear stresses along the cross section.

Tab. 3 Summary of results of individual analytical methods.

Analytical method	F_{mean} [kN]	V_{Rxz} [kN]	V_{Ryz} [kN]	V_{xz} [kN]	V_{yz} [kN]	τ_{xz} [MPa]	τ_{yz} [MPa]
M1	381.12	244.94	233.10	207.06	174.06	1.774	1.864
M2		362.90	304.61	173.92	207.20	1.596	1.902
M3		395.30	269.25	173.92	207.20	1.465	2.151
M4		388.82	261.09	173.92	207.20	1.490	2.219

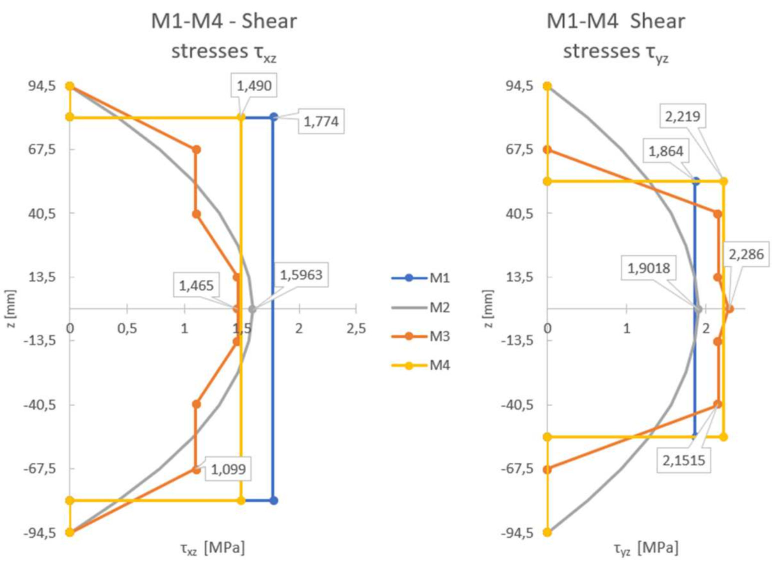

Fig. 7 Shear stresses along the cross section of CLT in direction *x* and *y*.

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The comparison between experimental resistance and resistance calculated using methods M1 to M4 reveals that the actual resistance exceeds the assumed resistance by up to 1.635 times in the case of M1 and 1.251 times for method M2, as shown in Tab. 4. Tab. 5 shows the sum of support reactions in each direction of CLT.

Tab. 4 Comparison to reference experiment.

Analytical method	F_{mean} **[kN]**	F_{max} **[kN]**	$\frac{F_{mean}}{F_{max}}$ **[-]**
M1	381.12	233.10	1.635
M2		304.61	1.251
M3		269.25	1.415
M4		261.09	1.460

Tab. 5 Sum of support reactions on FEM model of slab.

Sum of support reactions for direction i	
V_x **[kN]**	V_y **[kN]**
175.4	205.8

4 DISCUSSION

The results indicate that failure occurs in the secondary load direction first, which is consistent with the experimental observations [1].

A significant difference in shear forces for the different directions i can be observed. This difference arises because method M1 relies solely on empirical measurements from a parametric study based on the shear analogy method, conducted on a limited sample with constrained parameters. Mestek's analysis immediately assumes $0.67.n^{-1}$ fraction of total force is distributed to the major axis of CLT, without considering the span ratios. This can lead to significant inaccuracies. On the other hand, methods M2 to M4 take into account flexural stiffness and the corresponding spans in the distribution of shear forces. The accuracy of force distribution according to models M2 to M4 was confirmed using a simple FEM model, as shown in Figure 5. From these results it is clearly visible that methods M2 to M4 show good agreement with simplified analysis, and correctly predict higher shear force in the direction of minor axis of CLT slab and its values. The validity of Muster's idea of shear force distribution was also proven in [2] and [8].

From the results of shear stresses we can see that the Mestek's simplification of constant shear stress along the effective cross section is very close (see results of M4 and M3; M1 can be neglected due to wrong assumption of shear force) to direct calculation of shear stress via Grashof's equation. By this approximation, Mestek's method has a deviation of 1.7% in major axis and 3.1% in minor axis of CLT slab. This agreement must be confirmed on a larger number of samples. Method M2 does not describe the stresses on cross section accurately, the notion of homogenous rectangular cross section cannot be used when analysing shear stresses on CLT.

The underestimation of panel capacity in compression ranges from 1.25 to 1.64 times the experimental value. The most accurate was method M2 with deviation of 25%, despite the incorrect assumption of shear stresses.

5 CONCLUSION

By comparing these calculation methods, it is demonstrated that none of the currently known methods can accurately describe the failure of point-supported CLT panels under concentrated loading. The underestimation of panel capacity in compression ranges from 1.25 to 1.64 times the experimental value.

Methods M1 and M2 have many differences. The force distribution according to Muster (M2) shows good agreement with the FEM model, while Mestek's method is imprecise. Muster's method can be used to calculate the shear force on each side of the support when regular column grid is assumed.

The calculation of shear stress in M2 is based on a simplified Grashof formula, which assumes a parabolic stress distribution on a homogeneous section, which is not the case of CLT. Moreover, the calculation of shear stress for the x and y directions uses the same input data except for the force, so different cross-sectional characteristics for each direction are not considered. In Mestek's approach to calculating shear stress, an approximation of constant shear stress over the effective height is done. After further parametrical studies confirm the small deviation from Grashof's formula, the simplified method could be used to determine the rolling shear stresses. However, to achieve exact results, Grashof's formula should be used in its non-simplified form.

The rolling shear strength in interaction with compression perpendicular to grain near the support can be increased, according to Muster, by a factor of 1.6 for internal columns and 1.3 for external columns, which he demonstrated experimentally in his work. However, despite this, the samples fail at higher rolling shear stress values. A convincing conclusion regarding the increase in rolling shear resistance under concentrated loading has not been reached yet.

Further studies must be performed to balance the simplified analytical method in order for it to provide accurate and economical results.

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