PLACEMENT OF SHEAR STUDS ON A STEEL-CONCRETE COMPOSITE BEAM

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Abstract
The suitable placement of shear studs on a beam affects the quantity of studs required and their effective use. In practice, two different centre-to-centre spacings of coupling elements are most commonly used, with the denser placement near the supports where the shear force is greater. This paper investigates whether the use of three spacing sizes has a significant effect on reducing the number of studs needed, or whether the difference is almost negligible. At the same time, an attempt is made to achieve a change of spacing at the most favourable position on the beam so that the least number of shear studs can be used.

Keywords
Steel-concrete composite beam, theoretical research, layout, shear studs, centre-to-centre spacings

1 INTRODUCTION

Steel-concrete composite structures are most often used to bridge larger spans or in the construction of multi-storey buildings. They take advantage of the favourable properties of both materials to form effective and economical cross-sections. The design of composite structures is governed by the current Eurocode 4 [1], and specifically steel-concrete composite structures in accordance with Eurocodes 2 and 3 [2], [3].

Coupling elements have a major influence on the effective interaction between steel and concrete. Shear connectors have undergone considerable development over the existence of composite structures, from the now historical loops and blocks, which are almost unused today, to linear shear coupling and probably the most widely used shear studs with heads. These have the advantages of easy and quick connection and identical properties in all directions. An overview of the different types of coupling elements has been compiled by, e.g., Pardeshi and Patil from Sadar Vallabhbhai National Institute of Technology in Surat in India [4]. The effectiveness of coupling elements is affected not only by their material and dimensions but also by their placement on the beam. Much of the research on steel-concrete composite beams is focused on optimizing their dimensions and inventing new types of shear connectors, and the effect of their location is often overlooked. Research focusing on the location of shear studs in favourable and unfavourable positions in a concrete rib perpendicular to a steel beam, and their transverse spacing, has been carried out by a team from universities in the United Kingdom [5]. Due to the central stiffening rib in the trough of the profiled sheet, eccentric positioning of the shear studs was required, where the favourable position is where the concrete zone under compression in front of the stud in its load bearing direction is larger than the zone behind the stud. Conversely, the unfavourable position is one where the concrete zone in compression is smaller in front of the stud than behind it. For both positions, the transverse spacing was changed from 40 to 400 mm. The team found that the strength of the shear studs in the favourable position proved to be greater than in the unfavourable position, and that the resistance of the studs remains unchanged for transverse spacings of less than 80 mm and greater than 200 mm. The effect of coupling element spacings on fatigue stress was investigated in research at the King Fahd University of Petroleum and Minerals [6]. It has been clearly demonstrated that not all the spines on the beam are used to their maximum potential. Therefore, a redistribution of the shear studs where the minimum number of studs is used in the central part between the load points is more advantageous. Research carried out at universities in Iraq [7] focused on 4 different configurations of coupling element distribution, these being: shear studs located in two rows, in one row distributed along the longitudinal axis, single- and double- shear connectors distributed along the longitudinal axis, and in one row arranged staggered along the longitudinal axis. These configurations were all investigated for the following different spacings: 65, 85, 105, 150, 200 and 250 mm. The results from ABAQUS software proved that the configuration with two shear studs in a row has the best bending strength capacity, while the lowest capacity was for the single and double elements placed in alternation. The ideal spacing was found to be 105 mm in all groups. An investigation of the effect of longitudinal spacing on the coupling elements was also carried out at Kunming University of Science and Technology in China [8]. Here, they investigated the effect of deformation and slip on a two-pole beam as a function of the spacing of the coupling elements for two different spacing sizes and compared them with results obtained with uniformly spaced studs.

In practice, a maximum of two different spacings of coupling elements on a beam are commonly used. The reason for this is that too many different spacings would place higher requirements on workers and time. The question remains whether the use of three spacings would effectively reduce the number of studs, or whether the difference in the number required would be negligible. This research aims to follow on from previous work [9], where an attempt was made to determine the best position on a beam for changes to shear stud spacings so that a minimum number of studs can be used. Similarly, the most advantageous position for changing a total of three different spacings was found and compared with a beam with only two sizes of axial spacings of shear studs.

2 METHODOLOGY

The considerations in this research are based on previous work [9]. The plastic calculation does not include the actual load on the beam but assumes its maximum load-bearing capacity. Since the actual shear force, which varies along the length of the beam, is not considered here, the spacing of the shear force cannot be modified and uniform stud distribution is assumed. On the other hand, in the elasticity calculation, the actual shear force acting on the steel stud is taken into account and is assumed to be equal to the increase in the normal force in the concrete slab, which can be written as

$$V_{L,el} = \frac{V_{Ed} \cdot S_c \cdot s_l}{n \cdot I_i} \tag{1}$$

where V_{Ed} is the acting force, S_{c} is the first moment of the concrete slab, s_{l} is the centre-to-centre spacing of the shear studs, $n = E_{\mathrm{a}} / E_{\mathrm{c,eff}}$ is the modular ratio and I_{i} is the second moment of the ideal cross-section.

By arriving at the maximum force at the beam (this is the force above the support in the case of a simply supported beam), the value $s_{\mathrm{l,min}}$ is obtained from relation (1), which gives the largest possible centre-to-centre spacing of shear studs corresponding to the maximum shear force on the beam. If the maximum shear force V_{max} corresponds to the spacing of the coupling elements $s_{\mathrm{l,min}}$, then at any point on the beam distant from the support by a length x, the force $V_{(x)}$ will correspond to the spacing of shear connectors $s_{\mathrm{l},(x)}$. Similarly, at another point distant from point x by a distance y, the shear force $V_{(y)}$ will correspond to the spacing of shear studs $s_{\mathrm{l},(y)}$.

For a simply supported beam loaded with a uniform continuous load, the value of the maximum shear force is

$$V_{max} = \frac{1}{2} qL \tag{2}$$

where q is the value of the uniformly distributed load on the beam and L is the span of a simply supported beam.

Then, for the shear force at distance x from the support, the equation is

$$V_{(x)} = \frac{1}{2} qL - qx \tag{3}$$

where q is the value of the uniformly distributed load on the beam, L is the span of a simply supported beam and x is the distance from the support to the point on the beam where the centre-to-centre spacing of coupling elements first changes.

For the shear force $V_{(y)}$ at a distance y from point x toward the centre of the beam, the following relationship applies:

$$V_{(y)} = \frac{1}{2} qL - q \cdot (x + y) \tag{4}$$

where q is the value of the uniformly distributed load on the beam, L is the span of a simply supported beam, x is the distance from the support to the point on the beam where the centre-to-centre spacing of coupling elements first changes, and y is the distance from point x toward the centre of the beam where the spacing of shear studs changes for the second time.

The total number of shear studs on the beam can be calculated as the sum of their number on each section. The number of coupling elements on each section is determined as the length of the section divided by the spacing used on that section. If we consider 3 different spacings of shear studs, where the change in the size of the spacing occurs at point x and y, as shown in Fig. 1, where the smallest centre-to-centre spacing is near the supports and the largest in the centre of the beam, then the total number of shear studs is obtained from the relation

$$\mathrm{n} = \frac{2x}{s_{l,min}} + \frac{2y}{s_{l,(x)}} + \frac{L - 2x - 2y}{s_{l,(y)}} \tag{5}$$

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where x is the distance from the support to the point on the beam where the centre-to-centre spacing of coupling elements first changes, y is the distance from point x toward the centre of the beam where the spacing of shear studs changes for the second time, L is the span of a simply supported beam, $s_{l,min}$ is the axial distance of coupling elements corresponding to the shear force V_{max}, $s_{l,(x)}$is the axial distance of coupling elements corresponding to the shear force $V_{(x)}$ and $s_{l,(y)}$ is the axial distance of coupling elements corresponding to the shear force $V_{(y)}$.

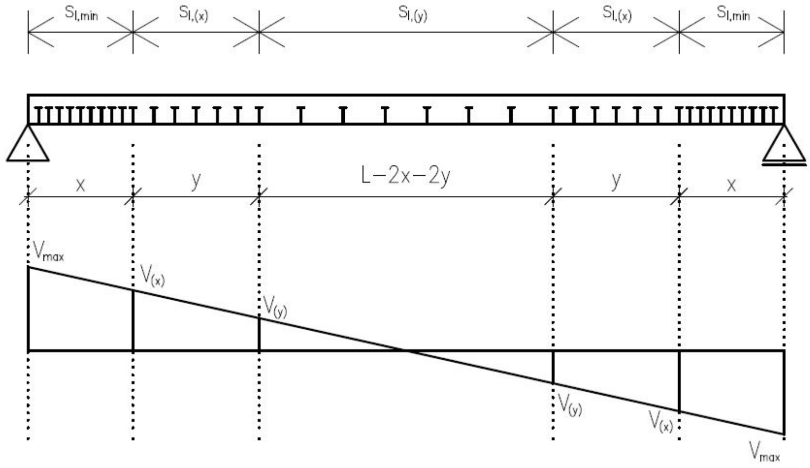

Fig. 1 Diagram of the shear stud distribution on a beam using three spacing sizes where the spacing change is at distance x and y.

By substituting equations (2), (3) and (4) and using their relationship to each other, the relation (5) can be further modified to give a new equation showing the effect of the number of shear studs on the beam depending on the change in the length of the individual sections x and y where the spacing of the coupling elements changes. This equation has the form

$$n = \frac{4}{s_{l,min} \cdot L} \cdot x^2 + \frac{4}{s_{l,min} \cdot L} \cdot xy + \frac{4}{s_{l,min} \cdot L} \cdot y^2 - \frac{2}{s_{l,min}} \cdot x - \frac{2}{s_{l,min}} \cdot y + \frac{L}{s_{l,min}} \tag{6}$$

which indicates that it is a quadric, specifically an elliptical paraboloid. From the shape of the elliptic paraboloid, it is clear that the minimum number of shear studs is reached at its vertex, as can be seen in Fig. 2. What is important is at what values of the coordinates x and y the vertex is reached. Since working in three dimensional space with quadrics is a little bit difficult, the problem has been simplified to the plane. This is because any cut of an elliptical paraboloid through a plane parallel to the tangent plane of the vertex of the paraboloid yields an ellipse for which the coordinates x and y of its centre are the same as the x and y coordinates of the vertex of the paraboloid.

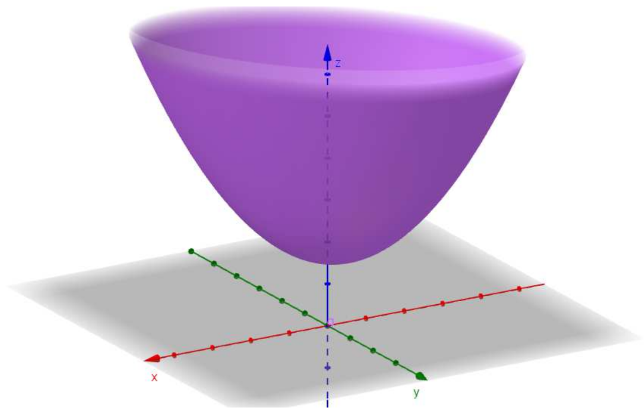

Fig. 2 The part of an elliptic paraboloid with the visible minimum value at the vertex (created with Geogebra [10]).

Setting equation (6) equal to zero gives an equation for the ellipse in the general position, i.e. its major and minor axes are not parallel to the axes of the coordinate system. Thus, a simple transformation to vertex form cannot be used to obtain its centre. From the value of the same coefficient for the x^2, y^2 and xy terms (or by using

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software such as Geogebra [10]), it can be found that the ellipse is rotated so that the major axis is in the second quadrant and makes a 45° angle with the axes of the coordinate system (so it has the equation $y = -x$). To get its centre, the ellipse would need to be transposed using transformation equations so that its axes are parallel to the axes of the coordinate system. Then, it would be necessary to transform it into a vertex form and then rotate the obtained vertex back by 45°. This procedure involves multiple steps and is quite long and laborious, but there is a faster variant based on the affine properties of conics described in chapter 14 in [11]. This says that the conic equation can be written as a matrix. In this case, equation (6) can be written matrix-wise as

$$\begin{pmatrix} \frac{4}{s_{l,min} \cdot L} & \frac{2}{s_{l,min} \cdot L} & -\frac{1}{s_{l,min}} \\ \frac{2}{s_{l,min} \cdot L} & \frac{4}{s_{l,min} \cdot L} & -\frac{1}{s_{l,min}} \\ -\frac{1}{s_{l,min}} & -\frac{1}{s_{l,min}} & \frac{L}{s_{l,min}} \end{pmatrix} \tag{7}$$

A central conic can be defined as any conic that is centre-symmetric according to each proper centre. Since an ellipse is centre-symmetric, it is a central conic and therefore must have a proper centre. According to [11], a system of equations converted to inhomogeneous coordinates can be used to calculate the centre of a conic (in our case, an ellipse). The required system for an ellipse based on the matrix (7) has the form

$$\begin{aligned} \frac{4}{s_{l,min} \cdot L} \cdot C_1 + \frac{2}{s_{l,min} \cdot L} \cdot C_2 - \frac{1}{s_{l,min}} = 0 \\ \frac{2}{s_{l,min} \cdot L} \cdot C_1 + \frac{4}{s_{l,min} \cdot L} \cdot C_2 - \frac{1}{s_{l,min}} = 0 \end{aligned} \tag{8}$$

By solving the system of equations (8), the coordinates of the centre of the ellipse can be found, and thus at the same time the x and y coordinates of the vertex of the elliptical paraboloid. For a given equation (6), the resultant vertex is

$$C\ [C_1; C_2] = \left[\frac{L}{6}; \frac{L}{6}\right] \tag{9}$$

where L is the span of a simply supported beam.

The coordinates show that the minimum amount of shear studs on the beam will be reached when the spacing is changed in length sections equal to 1/6 of the beam span, i.e. at 1/6 and 1/3 of the distance from the supports. This applies to all simply supported beams loaded with uniform continuous loads, regardless of their dimensions.

In reality, due to design principles, a limitation may occur if the value of the spacing corresponding to the force $V_{(y)}$ at a distance of 1/3 of the beam length from the support is greater than the permitted maximum spacings of shear studs $s_{l,max}$ (the smaller value of $6 \cdot h_c$ or 800 mm). In this case it is necessary to change the size of the centre-to-centre spacing of shear studs to the value $s_{l,max}$ before the distance of 1/3 L. The distance at which the force corresponding to the maximum allowed spacing of the coupling elements is achieved is reached at the distance x', which is obtained from the relation

$$x' = \frac{1}{2}\mathrm{L} - \frac{P_{Rd} \cdot n_r \cdot n \cdot I_i}{S_c \cdot s_{l,max} \cdot q} \tag{10}$$

where L is the span of a simply supported beam, P_{Rd} is the design load-bearing capacity of a shear stud, n_r is the number of shear studs in the transverse direction, $n = E_a / E_{c,eff}$ is the modular ratio, I_i is the second moment of the ideal cross-section, S_c is the first moment of the concrete slab, $s_{l,max}$ is the maximum allowed centre-to-centre spacing of shear studs and q is the value of the uniformly distributed load on the beam.

Therefore, if the limit of the design principles comes into force before 1/3 of the beam span, the question remains whether it is still suitable to change the first two spacing sizes at a distance of 1/6 of the beam length from the support. If in this research, for three spacing sizes, the ideal position for changing the axial distances of the shear studs is 1/3 of half the beam span, and in previous research [9] for two spacing sizes the ideal point on the beam was 1/2 of half the beam span, then it is possible that it may be most appropriate to divide the length x' (which does not use the $s_{l,max}$ spacing of shear studs) for two different axial distances of the shear studs in half. This will give two equal lengths of x (i.e. $x = y$), as shown in Fig. 3.

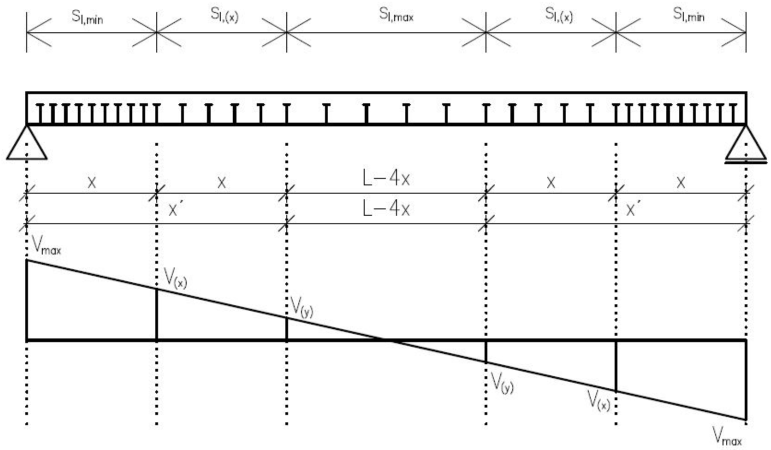

Fig. 3 Scheme of the shear stud layout configuration, where the maximum allowable stud spacing is applied along the entire length to which the limit of the design principles is relevant and the remaining edge parts are divided into two equal sections.

3 RESULTS

A simply supported beam of 10 m length loaded with a uniform continuous load of 15 kN/m was used as the base beam for the parametric study. The beam was formed from an IPE 300 profile made of S235 steel and a concrete slab of 80 mm height, with an effective width of 2200 mm made of C25/30 concrete. Shear studs with heads of strength 4.8, shank diameters of 16 mm and lengths of 50 mm were used as coupling elements.

The parametric study was performed using a spreadsheet, where the size of the *x* and *y* sections were varied in units of percent of the beam length (up to a maximum size $x + y = 50\%\ L$) and the total number of shear studs on the beam was calculated. This was done both for an ideal condition without limitation of the maximum axial distance of coupling elements by design principles, and a condition which includes the effect of design principles.

Finally, the results were also compared with the number of shear studs for the configuration of distribution where the section which is not affected by the influence of the design principles was divided into two equally sized parts.

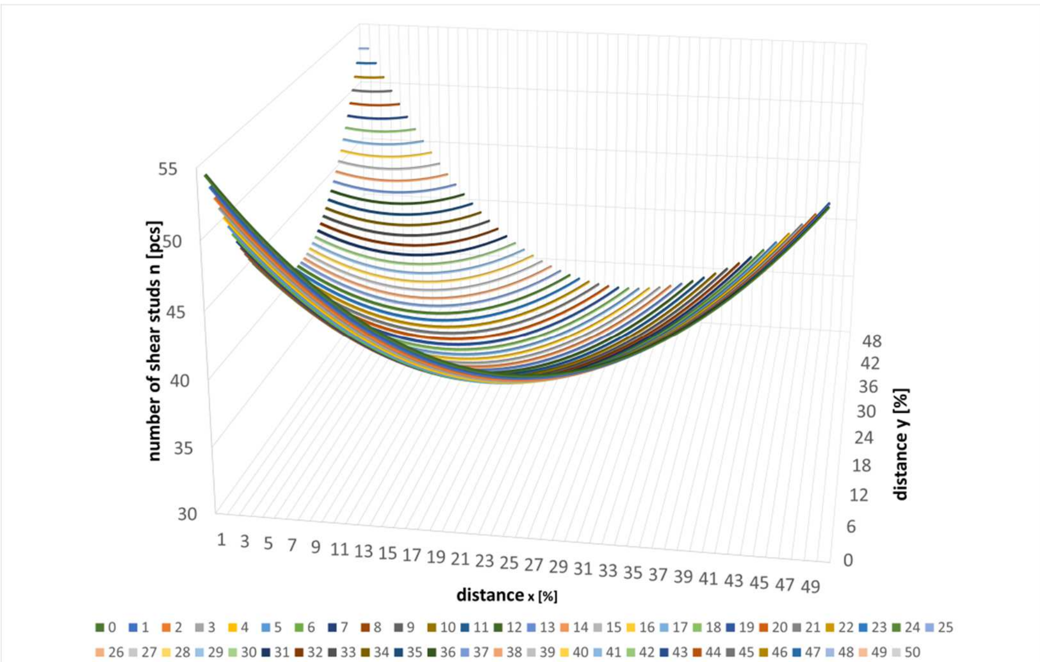

Fig. 4 Spatial graph of the dependence of the number of used shear studs on the length of the *x* and *y* sections in which the change of spacing was made. The graph is for the case without the limitations of the design principles.

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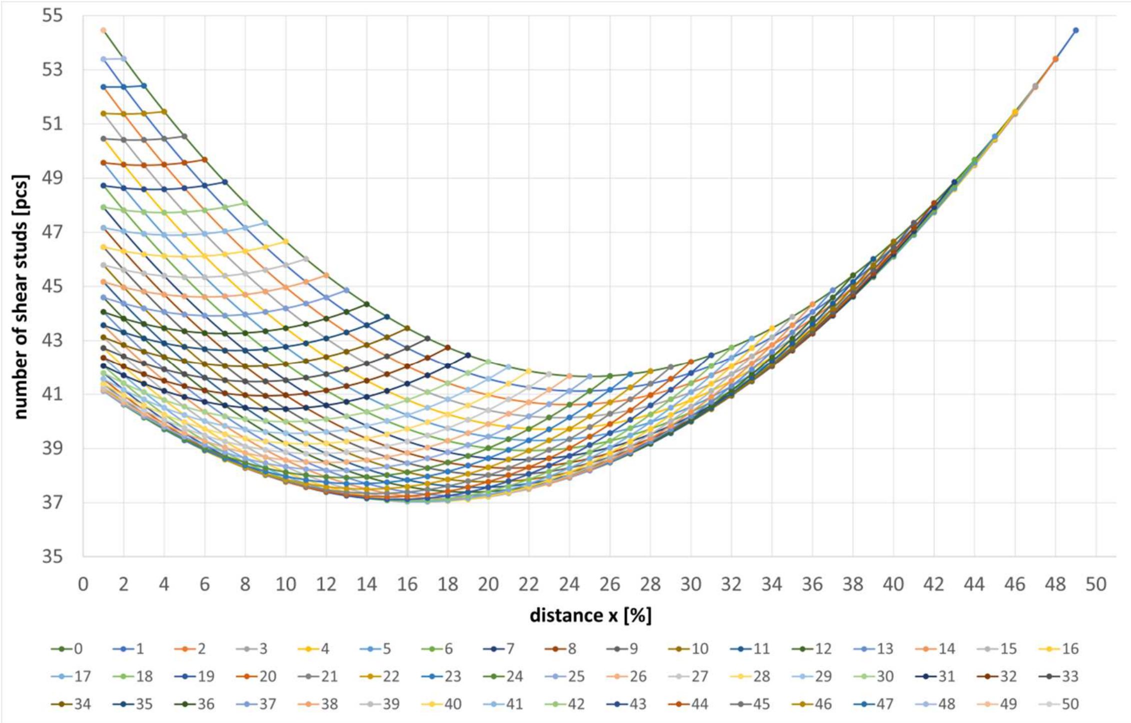

Fig. 5 Graph of the dependence of the number of shear studs on the percentage increase in the length of sections *x* for different lengths of section *y* (its percentage increase), for the ideal condition with no limitations from the design principles.

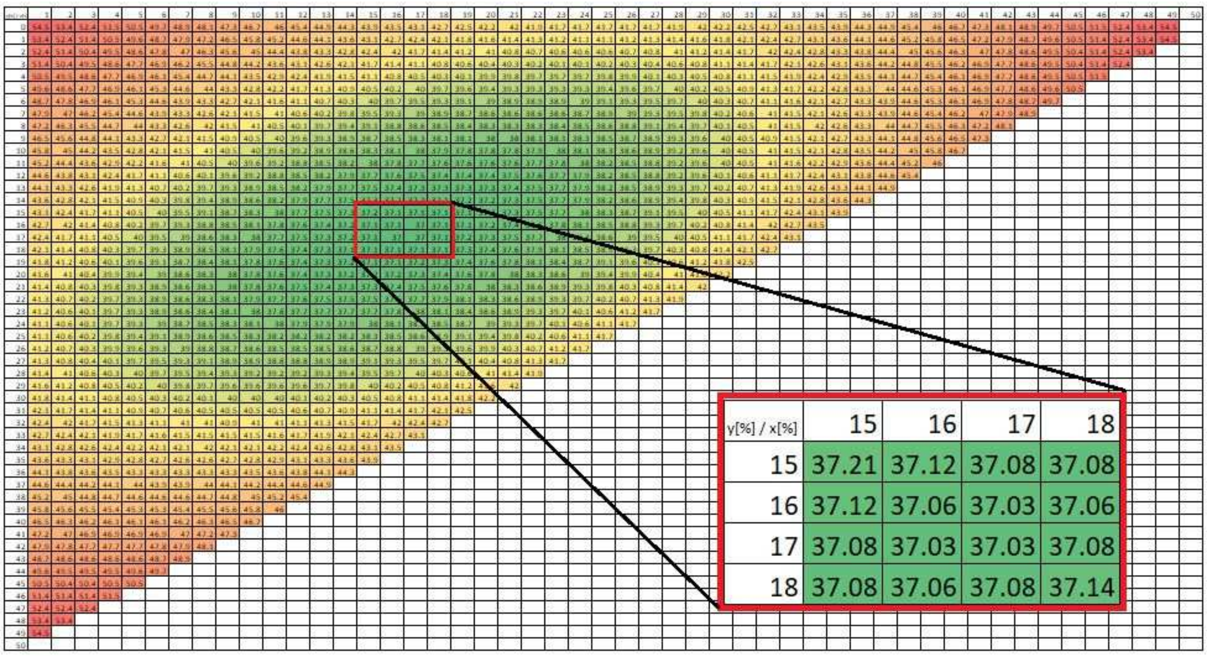

Fig. 6 A colour scale representing the amount of shear studs for percentage values of the lengths of the *x* and *y* sections from 1 to 49 for ideal conditions without the influence of limitations from the design principles, where green indicates the minimum number of studs and red the maximum.

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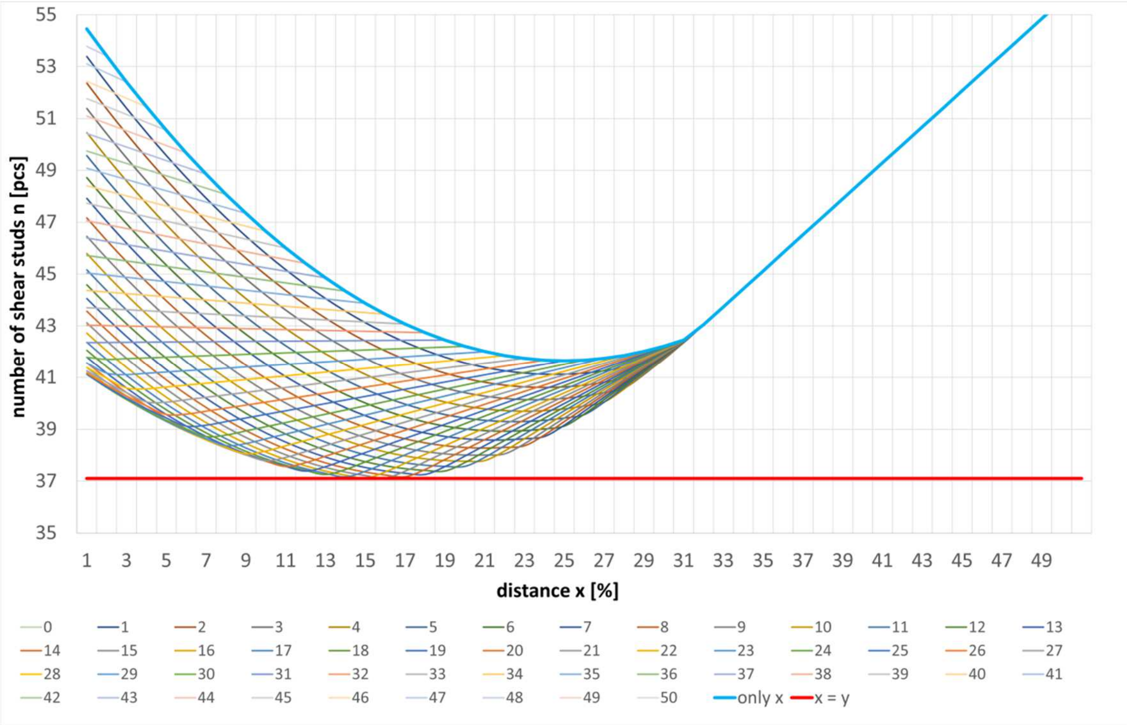

Fig. 7 Graph of the dependence of the number of shear studs on the percentage increase of the length of sections x for different lengths of section y (its percentage increase), including the influence of design principles. It includes the plotted results for the case with only two shear studs spacings (with only an x-section) and the case of distribution according to the configuration in Fig. 3.

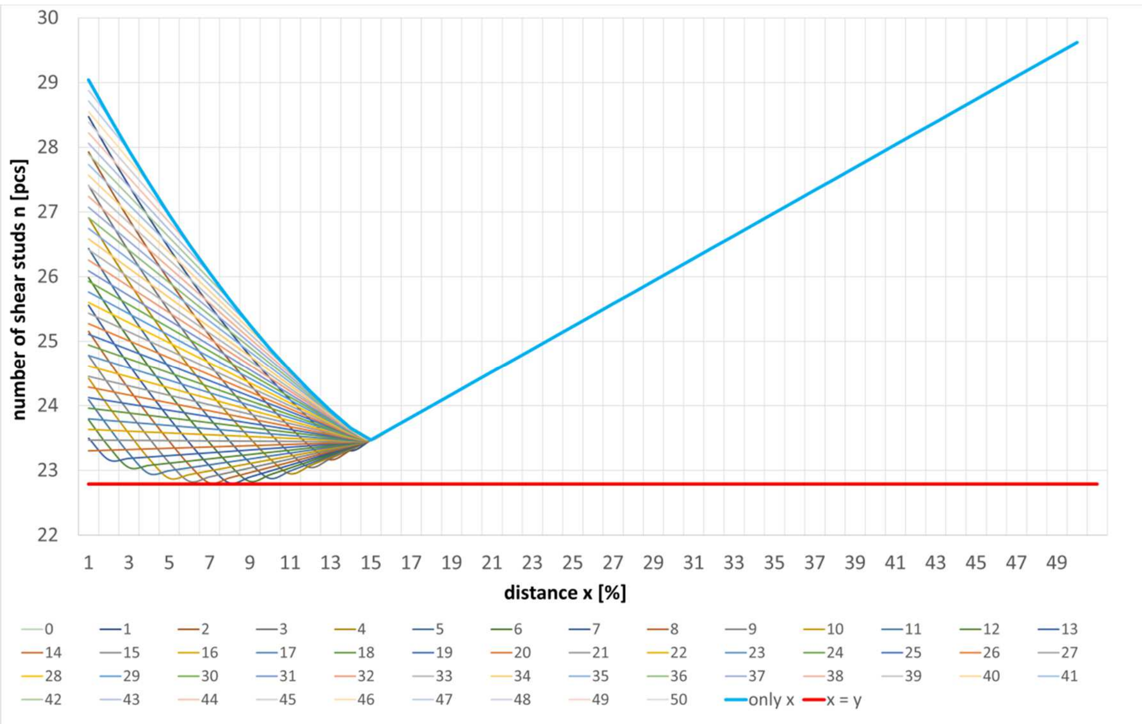

Fig. 8 Graph of the dependence of the number of shear studs on the percentage increase of the length of sections x for different lengths of section y (its percentage increase), including the influence of design principles. This is the case where the limit imposed by the design principles comes into effect before 1/6 of the beam span.

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4 DISCUSSION

The results of the parametric study show that when the number of studs depends on the length of the two segments x and y, the result is a function that indeed corresponds in shape to an elliptical paraboloid, as can be seen in Fig. 4. Thus, the correctness of the derived relation (6) was demonstrated, which was the starting point for further research. The minimum number of studs is reached just at the vertex of the paraboloid. Since it is harder to see from the graph in Fig. 4 at which size of the individual segments x and y the vertex is reached, the graph was transformed into a plane depending on the length of the segment x for each length of the section y, see Fig. 5. From Fig. 5 it can already be seen that the minimum is reached for the y curves corresponding to 16 and 17%, with the length of the x section corresponding to 16–17% of the beam span. This is equivalent to the results obtained from the coordinates of the centre of the ellipse (9), i.e. 1/6 L, which is 16.6% of the beam span. Thus, the ideal positions in which to make the change in spacing are always 1/6 of the span length from the support. These results can also be seen in Fig. 6, where the colour scale that underlines the values of the amount of coupling elements in the table is visible, with green for the minimum number of studs and red for the maximum. From the cutout of the table, it can be seen that the minimum is just reached when the x and y sections are between 16 and 17% of the beam span.

Fig. 5 shows the shapes of functions only if the maximum allowable spacing according to the design principles is not considered. If this limitation is taken into account, the constraint shown in the graph in Fig. 7 is obtained. It is already evident at first glance that there is a bending of the curves. This occurs when the maximum allowable axial distance of the shear studs is reached at the y-point or already at the x-point and its increase is no longer possible due to design limits. If this happens at the y point, the three spacing sizes are still used on the beam. The first $s_{l,min}$ corresponds to the shear force V_{max}, the second spacing $s_{l,(x)}$ corresponds to the force $V_{(x)}$ and the third is $s_{l,max}$ from the design principles. If the limit of the design principles were crossed at point x (on Fig. 6 at $x = 32$, where the curves join at one point and continue as a straight line), only two spacings would be used on the beam, namely $s_{l,min}$ and $s_{l,max}$. It depends on the particular case as to at which point on the beam the design already comes into effect. If the limit of the design principles comes into effect after the minimum, it is irrelevant to the search for the ideal length of the x and y segments and the change is best made at 1/6 L. In the opposite case, it may affect the ideal position of point x or y (as can be seen by comparing Fig. 7 and Fig. 8) and move it closer to the supports.

Next, a parametric study was used to verify the idea of using three spacing sizes on the beam by using the $s_{l,max}$ spacing over the entire length affected by the design principles (i.e. from the point x' calculated from relation (10)). Additionally, the remaining edge parts of the beam were divided into two equal sections, i.e. $x = y$. The obtained value of the number of coupling elements from this configuration of the arrangement was plotted on the graph in Fig. 7 as the red bold line. The results showed that the number of studs obtained from this arrangement corresponds to the minimum number of studs obtained from the procedure described above, even in cases where the design limit shifts the ideal points for changing the centre-to-centre spacing. It follows that if three spacing sizes are to be used on the beam, the ideal layout of studs is based on Fig. 3, when the distance from which the limit from design principles applies is calculated using equation (10) and the maximum allowable axial spacing of the shear studs $s_{l,max}$ is used on it. Also, the remaining parts are divided into two equal sections, with the section above the support using the spacing of the shear studs corresponding to the maximum shear force and the middle section using spacing corresponding to the force $V_{(x)}$. This method of using 3 different centre-to-centre spacings on a beam enables a smaller amount of coupling elements to be used than when only two different spacings are employed with a change in their size at the ideal point according to [9]. A comparison of the two methods can be seen in Fig. 7 (and Fig. 8), where the values for a beam with only two spacings (with only the x section) are plotted with a bold blue curve and the ideal method of distribution for three different spacings with a bold red line. The shear stud savings per beam depends on the individual case, but can be in the order of single digits per beam. It therefore depends on the situation, but especially for multi-storey buildings, even a saving of, e.g. 4 studs per beam will have a significant effect on the whole building.

5 CONCLUSION

The results proved the correctness of the derived relations for determining the ideal position for changing three different spacings of coupling elements on the beam. Under the ideal condition without any limitation due to design principles, it is ideal to change the spacing first at 1/6 of the span length from the support and second at 1/3 of the span length from the support. If the influence of design principles that may shift these positions is included, the smallest number of shear studs is obtained by using the stud distribution according to the configuration in Fig. 3. The maximum allowable stud spacing is used on the middle section that falls under the influence of design principles. The remaining edge parts are then divided into two equal length sections.

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This layout enables the use of fewer shear studs than when only two spacings are used. At the same time, the savings in studs is not negligible and, especially for larger structures, it can be an economically advantageous solution. The same principles that apply to headed studs could be applied to similar types of shear connectors. Research has only been carried out so far for simply supported beams with continuous uniform loading.

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