

SHEAR RESISTANCE OF PRESTRESSED BEAMS ACCORDING TO THE DIFFERENT DESIGN APPROACHES

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Abstract

The contribution deals with the shear resistance of prestressed beams and aims to compare experimental results with different design approaches. The experiment included two experimental tests on a sample prestressed beam. The course of the experiment is briefly described in the discussion. The prestressed beam, with an I cross-section, had bonded tendons and a total length of 7.0 m. A detailed description of the beam reinforcement, material characteristic and test setup are included. The reliability of individual design approaches is compared based on the analysis developed in this contribution.

Keywords

Shear, beam, prestress, I-cross section, bridge girder

1 INTRODUCTION

In Slovakia, there are approximately 1,800 prestressed bridges, the majority of which were built between 1963 and 2000. The advanced age of bridges also entails a decrease in prestressing, caused by material degradation, corrosion [1] or, in the case of accidents, damage [2]. Such degradation of the prestressing units reduces both the bending and shear [3] resistance of the cross-sections. The shear resistance of the cross-section has become the focus of several scientific studies [3], [4], [5]. The new second generation of Eurocodes [6] and the new edition of Model Code 2020 [7] have changed the design approaches to the shear resistance of reinforced concrete and prestressed members. The influence of the normal force on the shear resistance of the cross-section has increased compared to the first generation of Eurocodes [8] and approaches according to Model Code 2010 [9]. To verify the accuracy and reliability of the new generation of calculation approaches, experimental testing of prestressed reinforced concrete beams with an I-section shown in Fig. 1 is in progress. Predictions of the beams' resistance and their failure modes were presented in a paper at the Juniorstav 2023 conference [10]. These cross-sections are commonly used in practice, especially in the construction of bridge structures. The experimental testing contains 6 prestressed beams with two different shear reinforcement ratios and three different levels of prestressing. The basic geometry of the test setup is shown in Fig.1. This contribution works with data obtained from a pilot experiment of one prestressed beam. The results from a pair of tests performed on a prestressed beam are compared with values calculated based on computational approaches.

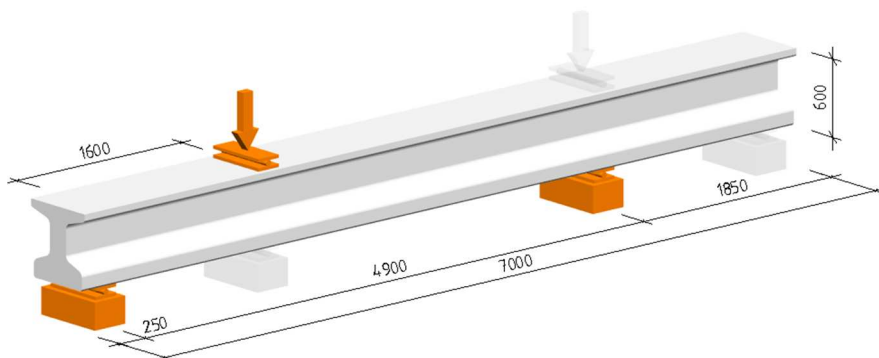


Fig. 1 The shape of the sample, the position of the supports and the applied force during experiment.

2 DETERMINATION OF SHEAR RESISTANCE ACCORDING TO SELECTED APPROACHES

Approach according to Eurocode EC 2 (2004)

The shear resistance of a prestressed beam according to Eurocode 2 (2004) [8] is evaluated by equation (1):

$$V_{Rd,s EC2} = \frac{A_{sw}}{s} \cdot z \cdot f_{wyd} \cdot \cot\theta \quad (1)$$

where A_{sw} is the shear reinforcement area in m^2 , s is the distance between stirrups in m, z is the lever arm for shear stress calculation in m, f_{wyd} is the design value of the yield strength of reinforcement in MPa, θ is the inclination of the compression field in $^\circ$.

The inclination of the compression field θ when determining the shear resistance was considered with a minimum value of $\theta_{min} = 22.0^\circ$.

Approach according to the second generation of Eurocode EC 2 (2023)

The shear resistance of a prestressed beam according to Eurocode 2 (2023) [6] is evaluated by equation (2):

$$\tau_{Rd} = \rho_w \cdot f_{wyd} \cdot \cot\theta \quad (2)$$

where ρ_w is the shear reinforcement ratio, f_{wyd} is the design value of the yield strength of reinforcement in MPa, θ is the inclination of the compression field in $^\circ$.

The new generation of Eurocode EC2 allows the use of the value $\cot\theta_{min} = 3.0$ for members subjected to significant axial compressive force (average axial compressive stress ≥ 3 MPa.), which represents the inclination of the compression field $\theta_{min} = 18.44^\circ$. For the purpose of comparing individual approaches, the shear resistance was recalculated according to equation (3):

$$V_{Rds FprEC2} = \tau_{Rd} \cdot b_w \cdot z \quad (3)$$

where τ_{Rd} is the design value of the shear stress resistance in MPa, b_w is the minimal width of the cross-section in m, z is the lever arm for shear stress calculation in m.

Approach according to Model Code 2020

Model Code 2020 [7] offers 3 levels of approximation for determining shear resistance. The first level of approximation represents a simplified conservative design and with an increasing level of approximation, more accurate results can be achieved, but they require the addition of more complicated parameters in the calculation. Shear resistance according to Model Code 2020 is defined by equation (4):

$$V_{Rd,s MC20} = \frac{A_{sw}}{s} \cdot z_v \cdot f_{wyd} \cdot \cot\theta \quad (4)$$

where A_{sw} is the shear reinforcement area in m^2 , s is the distance between stirrups in m, z_v is the lever arm for shear stress calculation in m, f_{wyd} is the design value of the yield strength of reinforcement in MPa, θ is the inclination of the compression field in $^\circ$.

The maximum shear resistance related to the crushing of concrete carrying the compression field is given in equation (5):

$$V_{Rd,max} = \frac{k_\varepsilon \cdot f_{cd} \cdot b_w \cdot z_v}{\cot\theta + \tan\theta} \quad (5)$$

where k_ε is the strength reduction factor, f_{cd} is the design concrete cylinder compressive strength in MPa, b_w is the minimal width of the cross-section in m, z_v is the lever arm for shear stress calculation in m, θ is the inclination of the compression field in $^\circ$.

1. Level of approximation – 1. LoA

The first level of approximation is considered with the following assumptions: The contribution of the concrete cross-section to the shear resistance is zero, $V_{Rdc} = 0$ kN. The inclination of the compression field is specified for a certain type of construction; in the case of prestressed members, it has a value of $\theta_{min} = 24.44^\circ$, which corresponds to $\cot\theta_{min} = 2.2$. The strength reduction factor has a fixed value $k_\varepsilon = 0.55$.

2. Level of approximation a) – 2. LoA a)

The second level of approximation, variation a), is derived on the basis of the strut-and-tie model. In this level of approximation, the inclination of the compression field takes into account the state of strain ε_x . This level also allows the use of the value $\varepsilon_x = 0.001$ or the calculation of the value of longitudinal strain according to cross-section stress. The value of the inclination of the compression field derived according to ε_x has value $\theta_{LoA2a} = 21.40^\circ$ in this case. The strength reduction factor k_ε takes a value based on ε_x and θ .

3. Level of approximation – modified 2. LoA a)

At this level of approximation, another parameter enters the calculation: β , representing the shear slenderness of the cross-section. The inclination of the compression field θ takes a value according to the inclination of β . The value of the cotangent θ should be in the range given by equation (6):

$$\cot\theta = \cot\beta + \sqrt{1 + \cot^2\beta} \leq \cot\theta_{min} \quad (6)$$

2. Level of approximation b) – 2. LoA b)

The second level of approximation, variation b), is derived based on the Modified Compression Field Theory. This variation of the approximation considers the shear capacity of the concrete cross-section together with the resistance of the shear reinforcement. The shear resistance of the concrete cross-section is given by equation (7):

$$V_{Rd,c} = k_v \cdot \frac{\sqrt{f_{ck}}}{\gamma_c} \cdot b_w \cdot z_v \quad (7)$$

where k_v is the reduction factor, f_{ck} is the characteristic concrete cylinder compressive strength in MPa, γ_c is the partial factor for concrete, b_w is the minimal width of the cross-section in m, z_v is the lever arm for shear stress calculation in m.

The total shear resistance of the cross-section according to Model Code 2020 and the second, respectively, the third level of approximation, is given in equation (8):

$$V_{Rd} = V_{Rd,s} \cdot V_{Rd,c} \leq V_{Rd,max} \quad (8)$$

The reduction factors k_v and k_ε relate to results even when mean values obtained from material tests were used. The reduction factor k_v works with the material properties of reinforcement, the geometry of cross sections and longitudinal strain ε_x . The reduction factor k_ε depends on longitudinal strain ε_x and the inclination of the compression field θ . The method of determining the value of k_v and k_ε varies depending on the degree of approximation.

3 DESCRIPTION OF THE TESTED SPECIMEN

The specimen is a prestressed reinforced concrete beam of I-cross section type DPS VP I/10. The total length of the beam is 7.0 m, while the effective span during the experiment is 4.9 m. The beam's length, the experimental scheme, and its support allow for two experimental tests on one beam. The cross-sectional characteristics of the prestressed beam are shown in Tab. 1. The standard reinforcement of the cross-section used in practice is supplemented with longitudinal concrete reinforcement of 6 bars with a diameter of $\phi 20$ mm. The shear reinforcement is adapted to the needs of the experiment and consists of stirrups with a diameter of $\phi 8$ mm in a grid of 125 mm. The degree of shear reinforcement is $\rho_w = 0.447\%$. The cross-section is prestressed by 10 bonded tendons with a diameter of $\phi 15.7$ mm (LSA), and the amount of prestressing is given in Tab. 2. The stress considered in the tendons at the time of the experiment is 1300 MPa. A scheme of the beam and the reinforcement of the cross-section is shown in Fig. 2. Material characteristics of concrete, reinforcement, and tendons are listed in Tab. 3. The distance of the applied force F from the support axis is 1600 mm. It is determined as three times the effective cross-section height $d = 547$ mm and provides a redistribution of the applied force to the supports in the ratio of 60% to 40%.

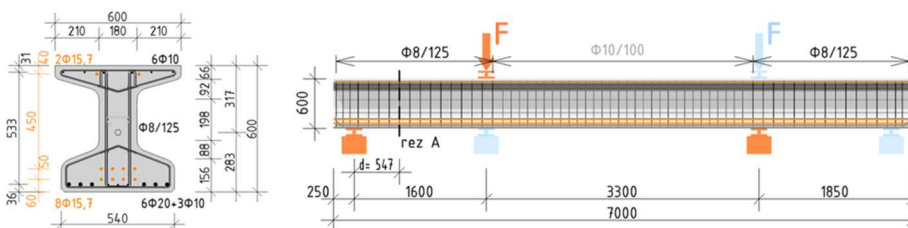


Fig. 2 A scheme of the beam and the reinforcement of the cross-section.

Tab. 1 Cross-section properties.

BEAM DPS VP I/10 – 600 mm			
Concrete cross-section area	A_c	0.20915	m ²
Ideal cross-section area	A_i	0.22466	m ²
First moment of area – A_c	S_c	0.0662623	m ³
First moment of area – A_i	S_i	0.07363	m ³
Second moment of area – A_c	I_{yc}	$8.6133 \cdot 10^{-3}$	m ⁴
Second moment of area – A_i	I_{yi}	$9.25613 \cdot 10^{-3}$	m ⁴
Shear reinforcement ratio	ρ_w	0.447	%

Tab. 2 Prestressing properties.

Level of prestress	Stress	Number of tendons	Prestressing force	
			1 tendon	10 tendons (P_{max})
[%]	[MPa]	[pcs]	[kN]	[kN]
100 %	1300	10	195	1950

Tab. 3 Laboratory properties obtained from laboratory tests.

Concrete			Reinforcement			Tendons		
			$\Phi 8$ mm	$\Phi 20$ mm				
$f_{c,cyl}$	[Mpa]	53.1	f_y	[Mpa]	560.7	Φ_p	[mm]	15.7
$f_{c,cube}$	[Mpa]	75.1	F_u	[Mpa]	625.0	f_p	[Mpa]	1806.2
E_c	[Gpa]	38.8	E_s	[Gpa]	200.0	f_{p01}	[Mpa]	1697.5
					E_p	[Gpa]	191.4	

4 DESCRIPTION OF THE TEST SETUP

The experimental testing was carried out in the central laboratories in March 2023. The experimental setup, as shown in Fig. 3, consisted of a test steel frame designed for testing beams with a capacity of 4.0 MN. A hydraulic press with a maximum output of 2.0 MN was attached to the redistributing steel beam. A load cell with a capacity of 2.0 MN was placed between the hydraulic press and the redistributing steel beam to measure the force applied to the beam. The transfer of force from the hydraulic press to the prestressed beam was ensured by a rounded steel element placed on the beam and a steel plate placed between this element and the hydraulic press. This load transfer method allowed for the rotation of the steel element together with the prestressed beam. The prestressed beam itself was supported by sliding supports on both sides during the experiment. These sliding supports consisted of a pair of steel plates between which a solid steel bar was inserted. The support closer to the applied force contained a load cell with a capacity of 2.0 MN to monitor the load redistribution during the experiment and to determine the shear resistance of the cross-section. Displacement of the beam was measured with LVDT sensors located in midspan, under the force and near the supports. Deformations and strains of the beam were measured with strain gauges glued to the surface of the beam. Also, one spring strain gauge was placed inside the beam and a pair of dynamometers was mounted on the tendons.

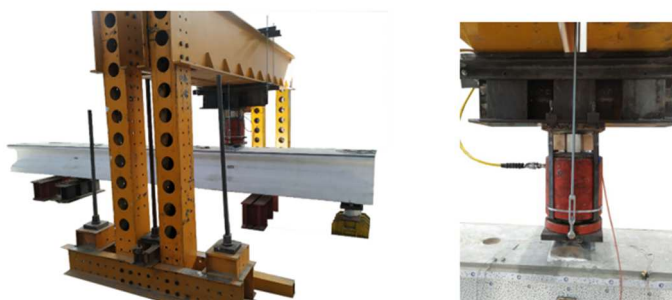


Fig. 3 The test setup (left) and the positioning of the hydraulic press (right).

5 ANALYSIS OF SHEAR RESISTANCE

In the analysis, the results of individual approaches to determining the shear resistance of prestressed members mentioned in Chapter 3 were compared. To eliminate the reliability factors of materials, the values considered in equations (1) to (8) were as follows, according to Tab.3(: $f_{ywd} = f_{yd} = f_y, f_{cd} = f_{c,cyl}, \gamma_c = 1,0$.

The calculated shear resistances according to the approaches were subsequently compared with the experimentally measured values, and these results are shown in Tab. 4. The comparison of shear resistances is shown in Fig. 4.

Tab. 4 Shear resistance of cross-section – $\rho_{sw} = 0.447\%$.

BEAM DPS VP I/10 – 600 mm – ϕ 8 mm / 125 mm => $\rho_w = 0.447\%$								
Experiment	Applied force F		Experiment		Average force in support			
	[kN]		Force in support		$V_{Rd,fin}$		$V_{Rtestm}/V_{Rd,fin}$	
			V_{Rtest}		[kN]		(-)	
N1.1	1200.3		765.8		796.7			
N1.2	1302.0		827.6					
Approaches								
LoA	V_{Rds}	V_{Rdc}	V_{Rd}	V_{Rdmax}	$V_{Rd,fin}$	$V_{Rtestm}/V_{Rd,fin}$		
	[kN]	[kN]	[kN]	[kN]	[kN]	(-)		
MC 2020								
I	481.5	-	-	\leq	961.5	\Rightarrow	481.5	165%
II a)	673.6	-	-	\leq	536.6	\Rightarrow	536.6	148%
III-II a)	1592.00	-	-	\leq	563.6	\Rightarrow	563.6	141%
II b)	673.6	101.9	775.5	\leq	726.0	\Rightarrow	726.0	110%
III-II (b)	563.6	101.9	665.5	\leq	854.4	\Rightarrow	665.5	120%
EC 2 (2004)	541.7					\Rightarrow	541.7	140%
EC 2 (2023)	656.6					\Rightarrow	656.6	121%

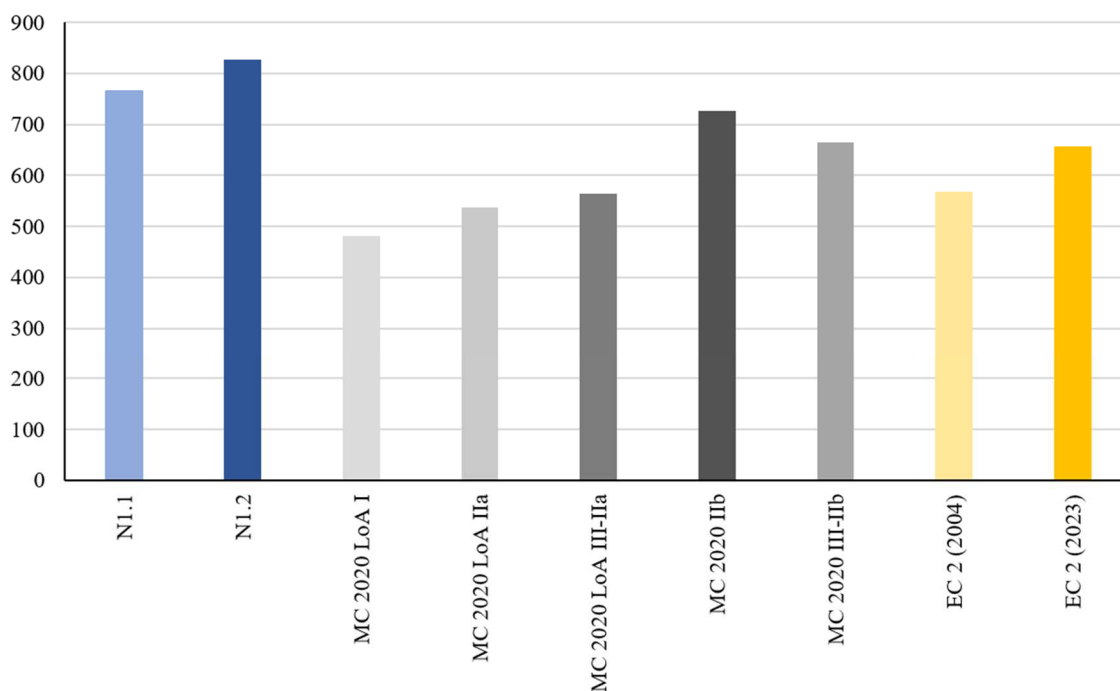


Fig. 4 Comparison of shear resistance according to approaches and experimental tests.

4 DISCUSSION

The tested specimen was loaded in steps of 50kN until a shear crack appeared, after which the loading step was changed to 25 kN. In both tests, the beam collapsed due to shear failure. The first shear crack appeared in both tests at a shear force of 420 kN, with a width of 0.20 mm for N1.1 and 0.25 mm for N1.2. The shear crack in the last step before failure reached a width of 2.5 mm in both tests. The shape and inclination of the shear crack after collapse are shown in Fig. 5 for test N1.1 and Fig. 6 for test N1.2. Based on the photos in Fig. 5 and Fig. 6 after the collapse of the beam, it is possible to determine the inclination of the shear crack at 24°.

All approaches can be considered safe because they did not reach the experimentally obtained values. In the second generation of Eurocode 2, by changing the inclination of the compression field from 22.00° to 18.44°, a reliability reduction of 19% was achieved compared to its first generation. The approach according to Model Code 2020 offers conservative solutions with sufficient reliability – at the first level of approximation up to 165%, but with a detailed analysis and the use of higher levels of approximation, the reliability of the approach compared to the experiment is reduced to 110–120%. The most accurate result of the compared approaches is achieved by Model Code 2020 at the second level of approximation of variation b) (110%), and at the third level of approximation of variation b) (120%). I attribute the accuracy of the results of Model Code 2020 variant b) to the addition of the resistances of the concrete part of the cross-section V_{Rdc} and the shear reinforcement V_{Rds} to the total shear resistance V_{Rd} . The split calculation of shear resistance separately for steel reinforcement and separately for the concrete cross-section can have an impact on more accurate results, especially for prestressed elements where the shear resistance of the concrete cross-section reaches significant values.



Fig. 5 Beam N 1.1 after collapse.

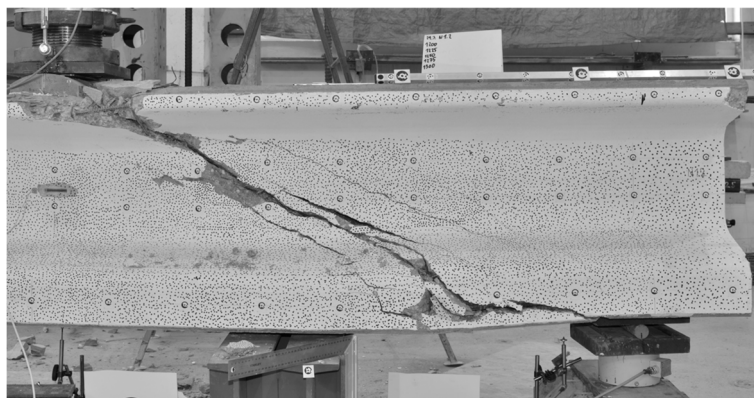


Fig. 6 Beam N 1.2 after collapse.

5 CONCLUSION

The shear resistance of the tested specimen was 768.8 kN in the test N1.1 and 827.6 in the test N1.2. The inclination of the shear crack in both tests was 24 °, which closely aligns with the values determined in the approaches.

Based on the analysis of the experiment results and the design approaches, it is possible to evaluate that the lowest degree of reliability of $V_{Rtestm} / V_{Rd,fin}$ is achieved by the approach according to the Model Code 2020 second (110%) and third (120%) levels of approximations of variants b) and the approach according to the second generation of Eurocode 2 (121%). After the end of the experiment on the remaining beams, the contribution will be extended with nonlinear analysis using software Atena GiD, including material nonlinearity.

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